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The Non-ideal Op Amp Method for Feedback Circuit Analysis

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Abstract A new approach to feedback circuit analysis called the non-ideal op amp method is proposed. The method is both accurate and simple to apply and solves the two main difficulties of the two-port analysis, namely the identification of the feedback type and the determination of the feedback network loading to the input and the output of the amplifier. The proposed methodology extends the standard op amp theory, treating every amplifier as a voltage amplifier, thus avoiding the problem of feedback type determination. All calculations refer to the unloaded open-loop circuit; therefore, there is no need to determine feedback loading. The article also demonstrates a technique for the correct calculation of the output impedance in current feedback.

Keywords: *feedback circuits, two-port analysis, return ratio, output impedance, loop gain*

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1. Introduction

The concept of feedback is fundamental in electronics and control systems. Feedback is the technique where a portion of the output is returned to the input. When the feedback signal is out of phase with the input the feedback is called degenerative or negative. Negative feedback has certain benefits, the most significant being the desensitization of the closed-loop gain. Other benefits include the extension of bandwidth, the reduction of noise and harmonic distortion. Negative feedback also modifies the input and output impedances, providing a means for tailoring the driving impedance at a specific port to our needs. The downside is a potential for instability that has to be taken care of at the design stage.

The analysis of feedback amplifiers, even with only a few components, is a complicated procedure because the feedback network loads the open-loop amplifier. In addition, both the amplifier and the feedback network cannot always be assumed to be unidirectional, especially at high frequencies.

Most textbooks present feedback theory in terms of two-port analysis [1-3], assuming unidirectional amplifier and feedback path. To analyze the circuit, one must first determine the type of feedback (voltage or current) and the type of signal summing at the input (series or shunt). Then the open-loop amplifier is drawn taking into account the loading that the feedback network presents to the input and output. The closed-loop gain is calculated from the open-loop gain *A* and the feedback factor *f*. A simplified analysis of feedback amplifiers based on the two-port methodology may be found in [4]. Similarly, Yeung's approach is essentially based on two-port analysis, [5]. Determining the type of feedback and the loading that results from the feedback network is not always a straightforward procedure. In addition, the calculation of the output impedance in current feedback using the two-port approach can give erroneous results.

Bode [6] developed feedback theory using the concept of the return ratio (*RR*). The *RR* for a controlled source can be found by setting all independent sources to zero, breaking the connection between the controlled source and the circuit, then driving the circuit at the break point with an independent source of equal strength and calculating the resulting output through the feedback loop. The technique was further refined by Rosenstark, [7]. Using Blackman's formula, we are able to find the impedance at any port, [8]. Few textbooks discuss the return ratio approach and not without a reason. Finding a dependent source that produces the simplest way to the result requires some experience. Otherwise, the procedure may be cumbersome and the result not particularly insightful.

Another technique for feedback circuit analysis is the one based on signal flow graphs, [9]. The method can in principle be used to handle any feedback architecture, however the choice of the parameters that represent each flow line is more or less an arbitrary process. Nicolic et al. [10], proposed a method that is based on the *RR* and the exact modelling of the amplifier without feedback. The resulting expressions for the loop gain and the *RR* are equivalent. Pellegrini [11], developed a new feedback theory based on the cut-insertion theorem. Another approach is the driving point impedance method introduced by Davis [12] and further developed by Ochoa

[13]. None of the above-mentioned techniques is particularly suited to undergraduate teaching.

In this paper, a general method for feedback circuit analysis is proposed based on the concept of the non-ideal op amp. The method treats all amplifiers as voltage amplifiers; hence, there is no need to determine the type of output sampling and input summation. The open-loop voltage gain, as well as, the input and output resistances refer to the unloaded amplifier and there is no need to calculate the loading that results from the feedback network. For quickness of calculations closed-loop expressions can be inserted in a spreadsheet. The non-ideal op amp methodology produces exact results as it does not make the assumption of unidirectional signal flow. The new method has been used in the class along with the traditional two-port technique and has been well received by the students.

The structure of the paper is as follows: In Section II the theoretical background for the analysis of both inverting and non-inverting feedback circuits is presented. Section III presents a number of examples, classified according to feedback type. In Section IV, an expression for the output impedance in current feedback is derived to complement the theory laid out in Section II. The paper closes by highlighting all significant contributions made to the field.

2. The Non-ideal Op Amp Method

2.1. Feedback Amplifiers

In a feedback design a large open-loop gain A is deliberately produced, then a portion f of the output is driven back to the input and subtracted from it to reduce gain to the desired value. In Figure 1 the arrows indicate that the signal flow is unidirectional, so there can be no signal transmission from the input to the output through the feedback network. Signals s_i, s_f, s_ε and s_o can be either voltages or currents. The output is directly proportional to the error signal s_e produced by subtracting the feedback signal s_f from the input.

$$s_o = As_\varepsilon = A\left(s_i - s_f\right) = As_i - fAs_o \tag{1}$$

From Eq. (1) the closed-loop gain is readily derived.

$$A_f = \frac{s_o}{s_i} = \frac{A}{1 + fA} \tag{2}$$

The gain A is reduced according to $1+fA$. If $A \to \infty$ the closed-loop gain A_f becomes $1/f$, i.e. equal to the reverse of the feedback factor and independent of the open-loop gain.

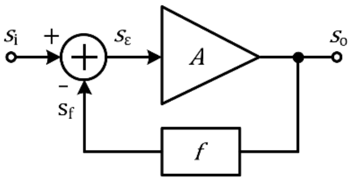


Figure 1. Block diagram representing a feedback amplifier with unidirectional signal flow where A is the open-loop gain and f the feedback factor

In two-port analysis, it is important to determine whether the signal sampled from the output is in the form of a voltage or a current. The first is said to be shunt sampling and the second series sampling. It is also necessary to identify if the feedback signal is added to the input loop in the form of a voltage (series mixing), or as a current inserted to the input node (shunt mixing). By taking all possible combinations feedback amplifiers may be divided into four categories:

voltage amplifiers with gain $A_{Vf} = v_o/v_i$
transconductance amplifiers with gain $G_{mf} = i_o/v_i$
transresistance amplifiers with gain $R_{mf} = v_o/i_i$
current amplifiers with gain $A_{If} = i_o/i_i$

Voltage feedback decreases R_o, while current feedback increases R_o. Series summing increases R_i, while shunt summing decreases R_i.

2.2. Non-inverting Amplifier

In the next paragraphs, we will derive expressions for the voltage gain and the driving point resistances at the input and output ports as modified by the application of feedback. To keep the analysis as simple as possible, we will ignore the source internal resistance and the load connected to the output. We will also assume that all parameters are frequency independent. Figure 2 shows the non-inverting op amp configuration. Voltage amplification is modelled with a voltage controlled voltage source with gain A; R_i is the input resistance and R_o the output resistance of the amplifier.

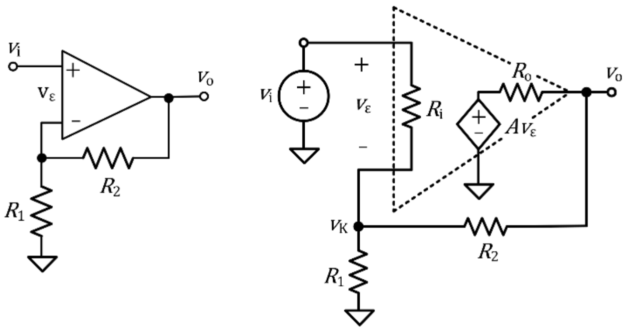


Figure 2. Left: Non-inverting op amp configuration with open-loop voltage gain A

The equations for node voltages v_K and v_o in matrix form are as follows:

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_i} & -\frac{1}{R_2} \\ \frac{A}{R_o} - \frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_o} \end{bmatrix} \cdot \begin{bmatrix} v_K \\ v_o \end{bmatrix} = \begin{bmatrix} \frac{1}{R_i} \\ \frac{A}{R_o} \end{bmatrix} \cdot v_i \tag{3}$$

Solving for v_o we find the expression for the closed-loop voltage gain

$$A_{Vf} = \frac{v_o}{v_i} = \frac{\left(R_1 + R_2\right) AR_i + R_1 R_o}{D} \tag{4}$$

where the denominator is

$$D = \left(A+1\right) R_1 R_i + \left(R_1 + R_i\right)\left(R_2 + R_o\right) \tag{5}$$

Usually, the term R_1R_o in the numerator is much smaller than the other term. Dividing the numerator and the denominator by $(R_1+R_2)R_i$ expression (4) is written in a more insightful form.

$$A_{Vf} = \frac{A}{\frac{R_2}{R_1+R_2} + f\left(A+1+\frac{R_o}{R_i}\right) + \frac{R_1 \| R_2}{R_i} + \frac{R_o}{R_1+R_2}} \quad (6)$$

where the operator || denotes the parallel combination of resistors and $f = R_1/(R_1 + R_2)$. All the terms that appear in the denominator cause a reduction in the open-loop gain. To get some idea of their relative contribution assume an amplifier with A = 300, R_i = 10 kΩ, R_o = 200 Ω, R_1 = 1kΩ, R_2 = 9 kΩ. The following table summarizes the contribution of each term in open-loop gain reduction. In a well-designed amplifier most of the reduction comes from the action of feedback, in other words from the term $f(A+1+R_o/R_i)$. Source internal resistance and the presence of an external load cause a further reduction in the open-loop gain.

Table 1. Percentage Reduction in Open-loop Gain

$\frac{R_2}{R_1+R_2}$	$f\left(1+A+\frac{R_o}{R_i}\right)$	$\frac{R_1\|R_2}{R_i}$	$\frac{R_o}{R_1+R_2}$
2.89%	96.75%	0.29%	0,07%

To find an expression for the closed-loop input resistance we express the ratio v_i/i_i as a function of the node voltages

$$R_{if} = \frac{v_i}{i_i} = \frac{v_i}{(v_i - v_K)/R_i} \quad (7)$$

Solving (3) for v_K and substituting to (7) we get

$$R_{if} = \frac{D}{R_1 + R_2 + R_o} \quad (8)$$

where D is taken from (5).

To derive an expression for the closed-loop output resistance we refer to the equivalent circuit of Figure 3, where the input has been connected to the ground and we have introduced the independent voltage source v_o that drives the amplifier output, producing a current i_o. Current i_o consists of two components

$$i_o = \frac{v_o - Av_\varepsilon}{R_o} + \frac{v_o}{R_1 \| R_i + R_2} \quad (9)$$

Using the voltage divider rule the error signal is written as

$$v_\varepsilon = -\frac{R_1 \| R_i}{R_1 \| R_i + R_2} v_o \quad (10)$$

Combining (9) and (10) we obtain the expression for the closed-loop output resistance

$$R_{of} = \frac{v_o}{i_o} = \frac{\left[(R_1+R_2)R_i + R_1R_2\right]R_o}{D} \quad (11)$$

where D is given by (5).

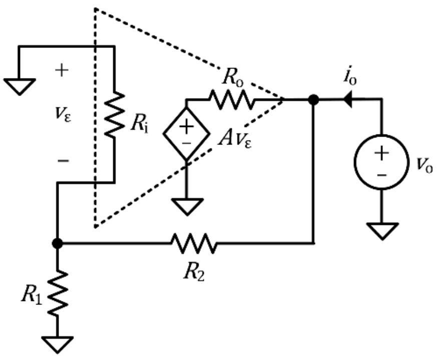
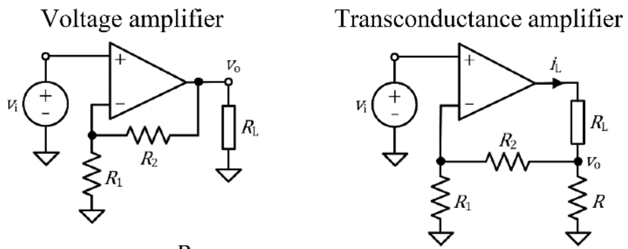


Figure 3. Equivalent circuit for the calculation of the closed-loop output resistance

Among others, two basic feedback circuits can be implemented with the non-inverting topology: the voltage amplifier and the transconductance amplifier. Figure 4 gives the expression for the ideal gain A_∞ (obtained when $A \to \infty$), as well as, the relation of the closed-loop voltage gain A_{Vf} to the desired closed-loop parameter.



$$A_\infty = 1 + \frac{R_2}{R_1} \qquad \frac{v_o}{v_i} = A_{Vf}$$

$$A_\infty = \frac{R + R_1 + R_2}{RR_1} \qquad G_{mf} = \frac{i_L}{v_i} \approx \frac{A_{Vf}}{R\|(R_1+R_2)}$$

Figure 4. Ideal gain and closed-loop parameter as a function of the closed-loop voltage gain

2.3. Inverting Amplifier

The inverting op amp configuration is depicted in Figure 5, along with its equivalent circuit. The parameters have the same meaning as in the previous paragraph. The node equations are written as

$$\begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_i} & -\frac{1}{R_2} \\ \frac{A}{R_o}-\frac{1}{R_2} & \frac{1}{R_2}+\frac{1}{R_o} \end{bmatrix} \cdot \begin{bmatrix} v_K \\ v_o \end{bmatrix} = \begin{bmatrix} \frac{1}{R_1} \\ 0 \end{bmatrix} \cdot v_i \quad (12)$$

Solving (12) for v_o we get the expression for the closed-loop gain

$$A_{Vf} = \frac{v_o}{v_i} = -\frac{(AR_2 - R_o)R_i}{D} v_i \quad (13)$$

where D is given by (5). In most practical amplifiers, the term R_o in the numerator is much smaller than AR_2 and can be omitted. Dividing the numerator and denominator by R_2R_i the closed-loop voltage gain can be written in the form

$$A_{Vf}=\frac{v_o}{V_i}=-\frac{A}{1+f\left(A+1+\frac{R_o}{R_i}\right)+\frac{R_1}{R_i}+\frac{R_o}{R_2}} \quad (14)$$

where $f = R_1/R_2$. In a properly designed amplifier most of the gain reduction comes from the action of feedback, i.e. from the term $f(A+1+R_o/R_i)$.

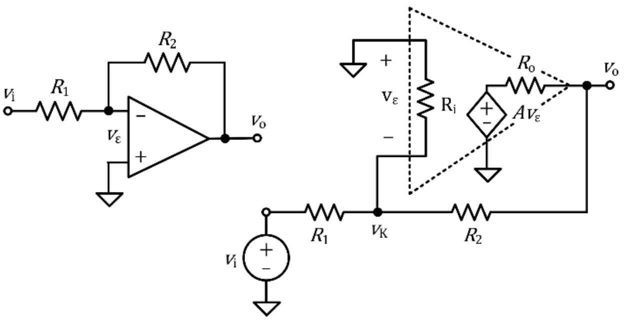


Figure 5. Left: Inverting op amp configuration. Right: controlled source model

The input resistance of the circuit is defined as

$$R_{if}=\frac{v_i}{i_i}=\frac{v_i}{(v_i-v_K)/R_1} \quad (15)$$

Solve (12) for v_K and substitute the result in (15) to get

$$R_{if}=\frac{D}{R_2+R_o+(A+1)R_i} \quad (16)$$

There is no need to calculate the output resistance, since the equivalent circuit is the same to that for the non-inverting amplifier (Figure 3). Expression (11) also holds for the inverting amplifier.

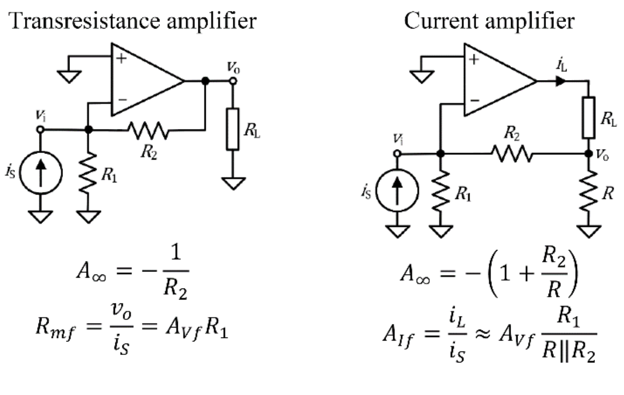


Figure 6. Ideal gain and closed-loop parameter as a function of the closed-loop voltage gain

Transresistance and current amplifiers are usually implemented using the inverting configuration. Figure 6 gives the expression for the ideal gain A_∞, as well as, the relation of the desired closed-loop parameter to the voltage gain A_{Vf} obtained with the non-ideal op amp method.

This concludes our analysis for the two basic amplifier configurations. Equations (4), (8), (11) form the basis for the analysis of amplifier circuits that do not invert their input, whereas equations (13), (16), (11) are used for inverting amplifiers. In the following paragraphs, a number of feedback circuits will be analyzed with the new methodology, classified according to the type of feedback and input summation.

3. Examples of Feedback Amplifiers Analyzed With the New Methodology

3.1. Series-shunt Feedback

In series-shunt feedback, the output voltage is sampled with a voltage divider and the feedback signal is subtracted in the input loop. The amplifier of Figure 7 is a typical case. Because of the asymmetry that exists in the differential stage inputs, the amplifier does not comply with the simple model of Figure 1 and the application of the two-port method is inappropriate. Neglecting feedback resistors R_1, R_2 the unloaded open-loop gain, input and output resistances are as follows:

$$A=g_{m1}r_{\pi5}g_{m5}r_o,\ R_i=2r_{\pi1},\ R_o=r_o \quad (17)$$

where g_m is the transistor transconductance, r_π its input resistance and r_o the total load at the collector of Q_5 excluding R_L. Using equations (4), (8), (11) we find the expressions for the closed-loop parameters indicated by the subscript "f".

$$A_{vf}=\frac{R_1r_o+2Ar_{\pi1}(R_1+R_2)}{(R_1+2r_{\pi1})(R_2+r_o)+2(A+1)r_{\pi1}R_1} \quad (18)$$

$$R_{if}=\frac{(R_1+2r_{\pi1})(R_2+r_o)+2(A+1)r_{\pi1}R_1}{R_1+R_2+r_o} \quad (19)$$

$$R_{of}=\frac{\left[R_1R_2+2r_{\pi1}(R_1+R_2)\right]r_o}{(R_1+2r_{\pi1})(R_2+r_o)+2(A+1)r_{\pi1}R_1} \quad (20)$$

with A taken from (17). These are exact expressions, not approximations. No assumption whatsoever has been made about the internal structure of the amplifier and signal flow. Note the common term that appears in all expressions resulting from the common term D in equations (4), (8), (11). This term is not identical to the $1+fA$ quantity of the two-port theory, because here A is the *unloaded* open-loop gain.

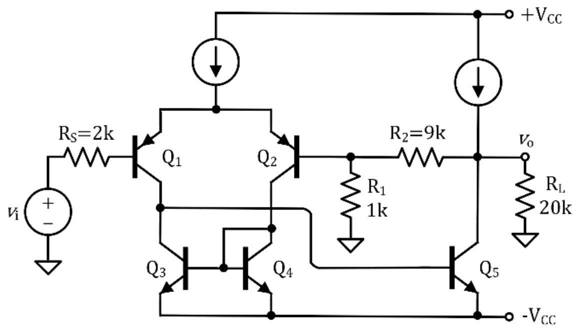


Figure 7. A two-stage voltage amplifier

Assuming $h_{fe} = 100$, $g_{m1} = 4$ mS, $r_{\pi1} = 25$ kΩ, $g_{m5} = 40$ mS, $r_{\pi5} = 2.5$ kΩ, $r_o = 50$ kΩ we find $A_{vf} = 9.97$, $R_{if} = 16.7$ MΩ, $R_{of} = 24.4$ Ω. While the input and output resistances are a property of the amplifier itself, the gain from the input to output should account for the source internal resistance and also the external load connected to the output. Assuming R_s = 2 kΩ, R_L = 20 kΩ the total gain is computed as

$$\frac{v_o}{v_i} = \frac{R_{if}}{R_s + R_{if}} \cdot \frac{R_L}{R_{of} + R_L} \cdot A_{vf} = 9.96 \qquad (21)$$

All results are verified by SPICE simulation.

Another example of a voltage amplifier is taken from reference [14], Figure 6. Quoting from this source: "*This differs from the basic voltage-feedback structure in that the current flowing into the left-hand side of the feedback network is not the input current to the amplifier without feedback (which happens to be zero in the case of a FET); rather, it is the source (or drain) current of the first stage.*"

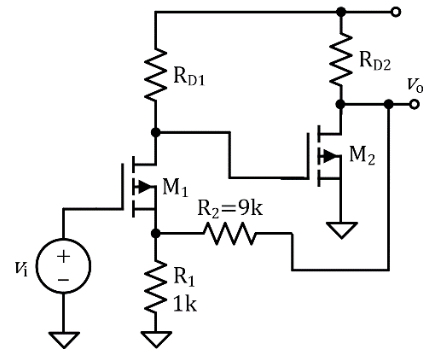


Figure 8. A two-stage voltage amplifier with FETs

With reference to the non-ideal op amp method, when feedback is applied to a point of low resistance, such as the emitter of a bipolar transistor or the source of a FET, a transformation of resistors is necessary, see [16]. For bipolars R_1, R_2, R_o should be multiplied by h_{fe}+1. Alternatively, R_i can be divided with h_{fe}+1 leaving R_1, R_2, R_o unchanged. In FETs the transformation factor is μ+1 where $\mu = g_m r_d$, where r_d is a high value resistor, usually the resistance "seen" when looking at the drain. For practical calculations, we will assume a value of 10^9 for both the drain and gate resistance. Dividing the input resistance by μ+1 gives

$$R_i = \frac{10^9}{g_m 10^9 + 1} \approx \frac{1}{g_m} \qquad (22)$$

After doing the calculations the value obtained for the closed-loop input resistance R_{if} should be multiplied by μ+1 (or h_{fe} +1).

Neglecting R_1, R_2 the open-loop voltage gain and output resistance of the amplifier of Figure 8 are

$$A = g_{m1} R_{d1} g_{m2} R_{d2}, R_o = R_{d2} \qquad (23)$$

From (4), (8) we get the expressions for the closed-loop parameters:

$$A_{vf} = \frac{g_{m1} R_1 R_{d1} + A(R_1 + R_2)}{(1 + g_{m1} R_1)(R_2 + R_{D2}) + (A+1) R_1} \qquad (24)$$

$$R_{of} = \frac{R_{d2}(R_1 + R_2 + g_{m1} R_1 R_2)}{(1 + g_{m1} R_1)(R_2 + R_{D2}) + (A+1) R_1} \qquad (25)$$

Assuming $g_{m1} = g_{m2}$ = 20 mS, $R_{d1} = R_{d2}$ = 5 kΩ, R_1 = 1 kΩ, R_2 = 9 kΩ we get: A_{vf} = 9.72, R_{if} = 686 GΩ, R_{of} = 92.3 Ω. SPICE simulation produces exactly the same results.

3.2. Shunt-shunt Feedback

In shunt-shunt feedback the output voltage is sampled and a current analogous to the output is injected to the input node. The closed-loop parameter $R_{mf} = v_o/i_i$ is called transresistance or mutual resistance. To apply the non-ideal op amp method to the inverting amplifier of Figure 9 the current source i_s along with its internal resistance R_s should be converted to their Thevenin equivalent, see right side schematic where R_s has been renamed as R_1. Then the open-loop parameters can be calculated by setting R_1 to zero and R_2 to infinity.

$$A = \frac{v_o}{v_i} = -g_m (R_c \| r_o) R_i = r_\pi R_o = R_c \| r_o \qquad (26)$$

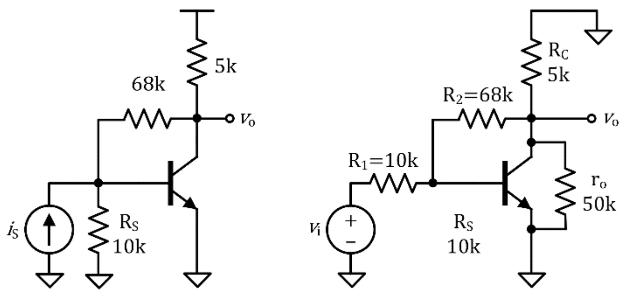


Figure 9. Single transistor transresistance amplifier

Using equations (13), (16), (11) the expressions for the closed-loop parameters are as follows

$$A_{vf} = -\frac{r_\pi R_c' (g_m R_2 - 1)}{(R_1 + r_\pi)(R_2 + R_c') + r_\pi R_1 (1 + g_m R_c')} \qquad (27)$$

$$R_{if} = \frac{r_\pi (R_2 + R_c')}{R_2 + r_\pi + R_c' + g_m r_\pi R_c'} \qquad (28)$$

$$R_{of} = \frac{R_c' [R_1 R_2 + r_\pi (R_1 + R_2)]}{(R_1 + r_\pi)(R_2 + R_c') + r_\pi R_1 (1 + g_m R_c')} \qquad (29)$$

where $R_c' = R_c \| r_o$. Equation (28) gives the expression for the closed-loop resistance at the base of the transistor after subtracting R_1. Assuming g_m = 40 mS, r_π = 2.5 kΩ, r_o = 50 kΩ, R_1 = 10 kΩ, R_2 = 68 kΩ we find A_{vf} = -5.64, R_{if} = 342.5 Ω, R_{of} = 726 Ω. The resistance that the current source sees is 342.5||10,000 = 331 Ω. To calculate the closed-loop transresistance we write

$$R_{mf} = \frac{v_o}{i_i} = \frac{v_o}{v_i / R_1} = A_{vf} R_1 = -56.4 k\Omega \qquad (30)$$

These results are verified by SPICE simulation.

3.3. Shunt-series Feedback

Shunt-series feedback is especially suited to current amplifiers. In the amplifier depicted on the left side of Figure 8 the output current is sampled with resistor R_2 and a portion of it is returned to the input node. To solve this circuit with the non-ideal op amp method we need to convert the current source to its Thevenin equivalent circuit. The output is the point where feedback is applied, therefore the open-loop voltage gain v_o/v_i ignoring R_1, R_2 ($R_1 = 0$, $R_2 \to \infty$) is

$$A_V = -\frac{g_{m1}r_o}{2}\cdot\frac{R+R_L}{1/g_{m3}+R+R_L}\cdot\frac{R}{R+R_L} \qquad (31)$$

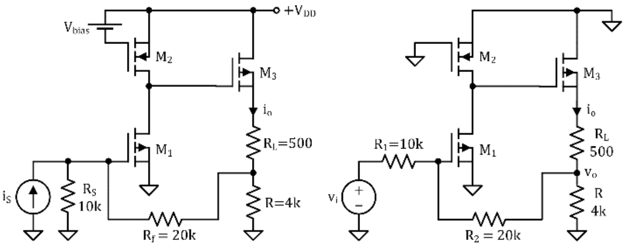


Figure 10. Left: current amplifier. Right: AC equivalent circuit with the current source transformed to a voltage source

The open-loop output resistance is $R\|(1/g_m+R_L)$. Assuming $g_m = 0.02$ S, $r_o = 50$ kΩ, $R_L = 500$, $R = 4$ kΩ, $R_1 = 10$ kΩ, $R_2 = 20$ kΩ we find $A = -440$, $R_o = 483.5$ Ω. For numeric computation purposes we will use $R_i = 10^9$ Ω. Using eqs. (13), (16) with $R_1 = 10$ kΩ, $R_2 = 20$ kΩ we get $A_{Vf} = -1.986$, $R_{if} = 46.5$. The resistance that the current source sees is 46.5||10,000 = 46.3 Ω. By making the approximation that the AC voltage at the gate of M_1 is zero the closed-loop current gain is computed as

$$A_{If} = \frac{i_o}{i_s} \approx \frac{v_o/(R\|R_2)}{v_i/R_1} = A_{Vf}\cdot\frac{10^4}{4\cdot10^3\|2\cdot10^4} = -5.96 \qquad (32)$$

SPICE simulation agrees with the above results and predicts an output resistance of 591.8 kΩ for the load. Obviously, the output resistance we have found is erroneous. The actual resistance that the load "feels" will be calculated in the next Section.

Equation (32) is a good approximation for the majority of practical circuits. An exact value for the current gain can be found if v_g is calculated from the formula

$$v_g = (R_1\|R_2)\left(\frac{1}{R_1}+\frac{A_{Vf}}{R_2}\right) \qquad (33)$$

The current gain is subsequently computed as

$$\frac{i_o}{i_S} = \frac{(v_o-v_g)/R_2+v_o/R}{(v_i-v_g)/R_1}. \qquad (34)$$

3.4. Series-series Feedback

In this type of feedback, a portion of the output current is fed back to the input and mixed with the input signal in the input loop. In the transconductance amplifier of Figure 11 the amplifier is assumed to have voltage gain $G = 1000$, input resistance $r_i = 10$ kΩ and output resistance $r_o = 500$ Ω. As mentioned in a previous paragraph the output is the point where feedback is taken from. The open-loop voltage gain neglecting R_1, R_2 is

$$A = \frac{R\cdot G}{R+R_L+r_o} = \frac{100\cdot1000}{100+1000+500} = 62.5 \qquad (35)$$

The open-loop input resistance is $R_i = r_i = 10$ kΩ and the output resistance $R_o = R \| (R_L+r_o) = 93.75$ Ω. From equations (4), (8), (11) we obtain the closed-loop parameters: $A_{vf} = 12.56$, $R_{if} = 47$ kΩ, $R_{of} = 19$ Ω. Because of the series feedback applied to the output, the R_{of} value is erroneous. The closed-loop transconductance is calculated as

$$G_{mf} = \frac{i_o}{v_i} = \frac{v_o/\left[R\|(R_1+R_2)\right]}{v_i} = \frac{A_{vf}}{R\|(R_1+R_2)} = 133.5mS \qquad (36)$$

SPICE simulation produces exactly the same results.

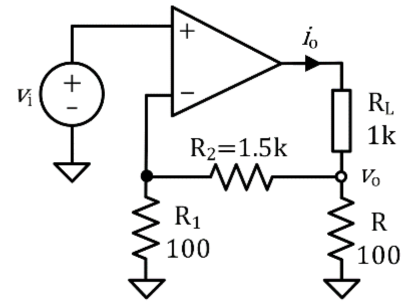


Figure 11. Transconductance amplifier drawn in op amp form

4. Output Impedance Calculation in Current Feedback

The output resistance as given by relation (11) is the resistance at the output node when the amplifier is operating in voltage mode. In series output feedback, sometimes called current feedback, we are interested to the resistance that the load "feels" due to the action of feedback. The higher the resistance the smaller is the current variation. The load maybe connected to the emitter or to the collector of the output transistor. We will start with the former case. A general equivalent circuit is shown in Figure 12. The output current is given by the expression

$$i_o = \frac{v_o+v_A-Av_\varepsilon}{R_o} \qquad (37)$$

where the error signal is computed as

$$v_\varepsilon = -\frac{R_1\|R_i}{R_1\|R_i+R_2}v_A \qquad (38)$$

At node A the following equation holds

$$\frac{v_A}{R}+\frac{v_A}{R_1\|R_i+R_2}+i_o = 0, v_A = -\frac{R(R_1\|R_i+R_2)}{R_1\|R_i+R_2+R}i_o \qquad (39)$$

Substituting (38), (39) to (37) and then solving for v_o/v_i we obtain the expression for the output resistance in series feedback.

$$R_{oE,series} = R_o + \frac{\left[R_2+(A+1)(R_1\|R_i)\right]R}{R_1\|R_i+R_2+R} \qquad (40)$$

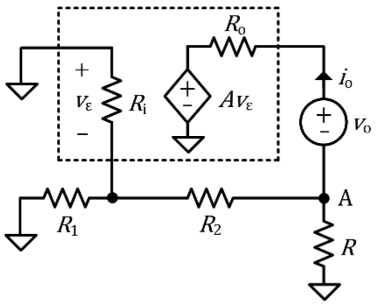


Figure 12. Equivalent circuit for the calculation of the output impedance when "looking" to the emitter

The resistance when looking down to the collector of the transistor (Figure 13) can be computed from the equation

$$R_{oC,series} = \left(1 + \frac{g_m r_\pi R_{oE,series}}{r_\pi + R_B + R_{oE,series}}\right) r_o \quad (41)$$

where R_B is the effective resistance at the base of the transistor. For FETs r_π is infinite and (41) simplifies to

$$R_{oD,series} = \left(1 + g_m R_{oS,series}\right) r_o \quad (42)$$

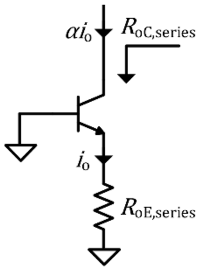


Figure 13. The resistance when looking down to the collector of the transistor. Resistance $R_{oE,series}$ results from the application of current feedback

Having derived equation (40) we are now able to calculate the output resistance "seen" by the load for the circuits of Figure 10 and Figure 11. For the circuit of Figure 10 (right side) the open-loop gain and output resistance are computed assuming infinite resistance connected at the source of transistor M_3. As usual we neglect resistors R_1, R_2. It is found that $|A| = g_m r_o/2 = 500$, $R_i \to \infty$, $R_o = 1/g_m = 50\ \Omega$. Using (40) with $R = 4$ kΩ, $R_1 = 10$ kΩ, $R_2 = 20$ kΩ we find $R_{oS,series} = 591.8$ kΩ which is also the value predicted by the simulation.

For the circuit of Figure 11, setting $R_1 = 0$ and disconnecting the amplifier output from the load, we get: $A = 1000$, $R_i = 10$ kΩ, $R_o = 500\ \Omega$. Then from eq. (40) with $R = 100\ \Omega$, $R_1 = 100$, $R_2 = 1.5$ kΩ we find $R_{of,series} = 6.42$ kΩ, a value that is verified by SPICE.

One last example to be examined is the transconductance amplifier of Figure 12 taken from [15]. In this work, it is mentioned that the two-port method fails to calculate the correct resistance at the collector of the transistor. For the amplifier depicted with the triangle we will assume a gain $G = 1000$, $r_i \to \infty$, $r_o = 0$. The bipolar transistor parameters are: $h_{fe} = 100$, $g_m = 40$ mS, $r_\pi = 2.5$ kΩ. We will first calculate the resistance seen at the emitter. For this reason, the inverting input is connected to the ground and the emitter is connected to a load with high AC impedance, for example an inductor with infinite self-inductance. The open-loop parameters are: $A = G$, $R_i = r_i$, $R_o = r_e$. Then using (40) with $R_2 = 0$ we get

$$R_{emitter} = r_e + (G+1)\left(R \| R_1\right) \quad (43)$$

In our case $R_1 \to \infty$ and (43) simplifies to

$$R_{emitter} = r_e + (G+1)R \quad (44)$$

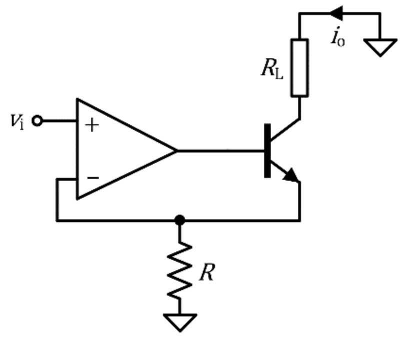


Figure 14. Calculation of the resistance presented to the load in a transconductance amplifier

Substituting the values given we get $R_{emitter} = 100.125$ kΩ. To find the expression for the resistance presented at the collector we use (41) with $R_B = 0$.

$$R_{collector} = r_o + \frac{g_m r_\pi r_o \left[r_e + (G+1)R\right]}{r_\pi + r_e + (G+1)R} = r_o + \frac{h_{fe} r_o R_{emitter}}{r_\pi + R_{emitter}} \quad (45)$$

Substitution of values gives $R_{collector} = 5.05$ MΩ which is exactly the value predicted by SPICE simulation. If the gain G is large the resistance at the collector becomes $R_{collector} = (h_{fe}+1)r_o$.

5. Discussion

The non-ideal op amp method has been proposed as a simpler and more accurate alternative to the established two-port methodology. It overcomes the two main difficulties of the two-port method, namely the identification of feedback type and the determination of feedback loading to input and output. The first difficulty does not exist because every amplifier is treated as a voltage amplifier and the second is overcome by calculating the unloaded open-loop quantities. The results obtained are exact; no assumption is made for the amplifier structure or the signal flow. If numerical results are wanted, equations can be inserted in a spreadsheet. This saves time and guarantees accuracy. If we need to have the full closed-loop expressions, we can Maple, Matlab symbolic toolbox or SymPy (a Python library for symbolic mathematics). The effort required is minimal. This is not possible to do with the return ratio analysis or other methods.

The non-ideal op amp method provides intuition on the loading caused to the open-loop gain by the various parameters, eqs (6) and (14). This way the student or even the experienced designer can find which parameter needs optimization.

The proposed method has been presented using a constant open-loop gain and resistances. However, there is no reason why it should not work with frequency dependent components or parameters. To this end, resistors can be substituted with impedances and the open-loop gain with a frequency dependent expression $A(f)$.

The steps taken to apply the proposed methodology are as follows:

i) find out if the amplifier inverts its input signal or not

ii) if a current source is connected to the input replace it with its Thevenin equivalent circuit

iii) identify feedback resistors R_1, R_2 and calculate the unloaded voltage gain, input resistance and output resistance

iv) when the feedback is returned to a point of low resistance, such as the emitter of a bipolar transistor or the source of a FET, a transformation of resistances is necessary.

iv) calculate the closed-loop parameters. For the non-inverting case use Eqs. (4), (8), (11). For the inverting case use Eqs. (13), (16), (11).

v) if necessary, express the desired closed-loop quantity as a function of A_{Vf} and the other circuit parameters.

The proposed method has been used in the class for a number of years as a tool for teaching feedback in undergraduate courses about analog electronics. The students found its application simpler than the other methods. Typically, the non-ideal op amp is used as a means to quickly get the correct result and then compare with other established methods that may offer more insight into the action of feedback.

6. Conclusions

The non-ideal op amp method proposed here builds upon the well-known theory of op amp circuits. It is a general method as it makes no assumptions about the structure of the amplifier or the direction of signal transmission. The proposed methodology treats every amplifier as a voltage amplifier. It is simple to apply, as only the unloaded quantities need to be calculated. This way the main difficulties in the application of the two-port methodology (identification of feedback type and loading from the feedback network) are inherently solved.

In addition to the non-ideal op amp approach, a methodology that allows the correct calculation for the output impedance in current feedback has been presented. The application of the two-port analysis is problematic in this situation and cannot distinguish between the cases where the load is connected to the emitter or to the collector of the transistor.

All expressions derived in this paper use resistors, however the proposed methodology also works for circuits with frequency dependent components. Work to calculate transfer functions using the non-ideal op amp method is underway.

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