American Journal of Electrical and Electronic Engineering, 2015, Vol. 3, No. 2, 37-43
Available online at http://pubs.sciepub.com/ajeee/3/2/3

DOI:10.12691/ajeee-3-2-3

Synchronization of Diffusively Coupled Oscillators: Theory and Experiment

B. Nana[1,*], P. Woafo[2]

[1]Department of Physics, Higher Teacher Training College, University of Bamenda, PO Box 39 Bamenda, Cameroon
[2]Laboratory of Modelling and Simulation in Engineering, Biomimetics and Prototypes, Faculty of Science, University of Yaounde I, PO Box 812 Yaounde, Cameroon
*Corresponding author: na1bo@yahoo.fr

Received February 24, 2015; Revised March 23, 2015; Accepted April 02, 2015

Abstract In this paper complete synchronization of diffusively coupled oscillators is considered. We present the results of both, theoretical and experimental investigations of synchronization between two, three and four almost identical oscillators. The method of linear difference signal has been applied. The corresponding differential equations have been integrated analytically and the synchronization threshold has been found. Hardware experiments have been performed and the measured synchronization error of less than 1% has been determined. Good agreement is found between theoretical and experimental results.

Keywords: *Chaos, Synchronization, Oscillator*

Cite This Article: B. Nana, and P. Woafo, "Synchronization of Diffusively Coupled Oscillators: Theory and Experiment." *American Journal of Electrical and Electronic Engineering*, vol. 3, no. 2 (2015): 37-43. doi: 10.12691/ajeee-3-2-3.

1. Introduction

In the last few years, several researchers have focused their attention on the problems related to the synchronization of chaotic systems [1-5]. Today the potential of chaos theory is recognized in the world-wide with research groups actively working on this topic [6-10]. One of the great achievements of the chaos theory is the application in secure communications. The chaos communication fundament is the synchronization of two chaotic systems under suitable conditions if one of the systems is driven by the other. Since Pecora and Carrol [11] have demonstrated that chaotic systems can be synchronized, the research in synchronization of couple chaotic circuits is carried out intensively and some interesting applications such as communications with chaos have come out of that research.

There are several methods for synchronizing chaotic oscillators described in literature [12-17]. The simplest one employs the feedback in the form of linear difference between the output of the transmitter $u_1(t)$ and the output of the receiver $u_2(t)$. The difference signal $K(u_2 - u_1)$ when applied with a certain weight $K > K_{\min}$ to an appropriate input of the receiver synchronizes the latter to the transmitter [16]. Although there are many papers describing global synchronization of a network of coupled oscillators, less attention has been devoted to experimental results for bidirectional coupled systems.

In this paper, attention will be drawn to complete synchronization of two, three and four coupled chaotic oscillators. The remainder of this paper is organized as follows. In section 2, we present the used oscillator and study its dynamic behavior as a function of control parameter. In section 3, synchronization of two, three and four diffusively coupled oscillators are analyzed theoretically. Section 4 deals with the experimental setup and where experimental results are compared to the numerical ones. Finally conclusions are drawn in section 5.

2. Circuit Model and Description

2.1. Circuit Model

The circuit diagram of the oscillator is shown in Figure 1.

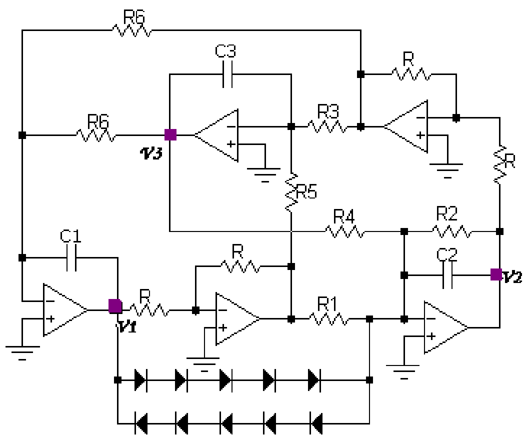

Figure 1. Electronic scheme of the oscillator

The circuit is a third order nonlinear oscillator containing five operational amplifiers. We assume that all the operational amplifiers operate in their linear domain. In our model, the diode acts like non linear component and we model its voltage-current characteristic with an exponential function, namely

$$i = I_0\left[\exp\left(\frac{v}{V_0}\right) - 1\right] \quad (1)$$

where i is the current through the diode, v is the voltage across the diode, I_0 is the inverse saturation current and $V_0 \simeq 26\,\text{mV}$ at the room temperature. The dynamics of the oscillator is given by the following set of differential equations:

$$\frac{dv_1}{dt'} = \frac{1}{R_6C_1}v_2 - \frac{1}{R_6C_1}v_3, \quad (2)$$

$$\frac{dv_2}{dt'} = \frac{1}{R_1C_2}v_1 - \frac{1}{R_2C_2}v_2 - \frac{1}{R_4C_2}v_3 - \frac{2I_0}{C_2}\sinh\left(\frac{v_1}{5V_0}\right), \quad (3)$$

$$\frac{dv_3}{dt'} = \frac{1}{R_5C_3}v_1 + \frac{1}{R_3C_3}v_2, \quad (4)$$

Introducing the following dimensionless variables and parameters

$$\begin{gathered} x = \frac{v_1}{5V_0},\ y = \frac{v_2}{5V_0},\ z = \frac{v_3}{5V_0},\ t = \omega_0 t',\ a = \frac{1}{R_1C_2\omega_0}, \\ b = \frac{1}{R_2C_2\omega_0},\ c = \frac{1}{R_4C_2\omega_0},\ d = \frac{1}{R_3C_3\omega_0} \quad (5) \\ \text{where } \omega_0 = \frac{1}{R_6C_1} \text{ and } 5V_0C_2 = 2I_0R_6C_1, \end{gathered}$$

we come to the set of differential equations convenient for numerical integrations

$$\dot{x} = y - z, \quad (6)$$

$$\dot{y} = ax - by - cz - \sinh(x), \quad (7)$$

$$\dot{z} = x + dy. \quad (8)$$

2.2. Dynamic behavior

Assuming that in the oscillatory state, variables x, y and z can be replaced by their corresponding virtual orbits x_0, y_0 and z_0 respectively, we find x_0, y_0 and z_0 in the following form:

$$\begin{cases} x_0 = P_1\cos\omega t - Q_1\sin\omega t \\ y_0 = P_2\cos\omega t - Q_2\sin\omega t \\ z_0 = P_3\cos\omega t - Q_3\sin\omega t \end{cases} \Rightarrow \begin{cases} x_0 = Ae^{j\omega t} + \overline{A}e^{-j\omega t} \\ y_0 = Be^{j\omega t} + \overline{B}e^{-j\omega t} \\ z_0 = Ce^{j\omega t} + \overline{C}e^{-j\omega t} \end{cases}$$

with (9)

$$A = \frac{1}{2}(P_1 + jQ_1),\ B = \frac{1}{2}(P_2 + jQ_2),\ C = \frac{1}{2}(P_3 + jQ_3).$$

Using the above expressions and neglecting the higher harmonics, we can show that

$$\begin{gathered} (x_0)^{2n+1} = \frac{(2n+1)!}{n!(n+1)!}|A|^{2n}x_0 \text{ then} \\ \sinh(x_0) \simeq \sum_{n=0}^{N}\left[\frac{(x_0)^{2n+1}}{(2n+1)!}\right] \simeq x_0\sum_{n=0}^{N}\left[\frac{|A|^{2n}}{n!(n+1)!}\right]. \end{gathered} \quad (10)$$

By substituting system (8) and equation (9) in the set of equations (5), (6) and (7) and equating the coefficients of $e^{j\omega t}$ and $e^{-j\omega t}$ separately to zero, we obtain the radian frequency of oscillatory orbit by. While the amplitude of oscillation is obtained by solving the following polynomial equation:

$$\sum_{n=0}^{N}\left[\frac{|A|^{2n}}{n!(n+1)!}\right] - r = 0 \text{ with } r = a + \frac{c(1-bd)}{b+d}. \quad (11)$$

In view to derive the analytical expression of amplitude A, we fixe $N = 3$ and we obtain the following result:

$$\begin{gathered} |A| = \left[-4 - 2\left(-1 - 9r + 3\sqrt{1 + 2r + 9r^2}\right)^{1/3} + \right. \\ \left. 2\left(1 + 9r + 3\sqrt{1 + 2r + 9r^2}\right)^{1/3}\right]^{1/2} \end{gathered} \quad (12)$$

This system presents stationary, periodic and chaotic attractors depending on the value of the parameters (a, b, c, d). The bifurcation diagram as well as the Lyapunov exponent drawn in Figure 2 show that for certain sets of parameters, the system exhibits chaotic oscillations. We used c as control parameter and other used parameters are the following: $a = 3.5$, $b = 0.5$, $d = 1.2$.

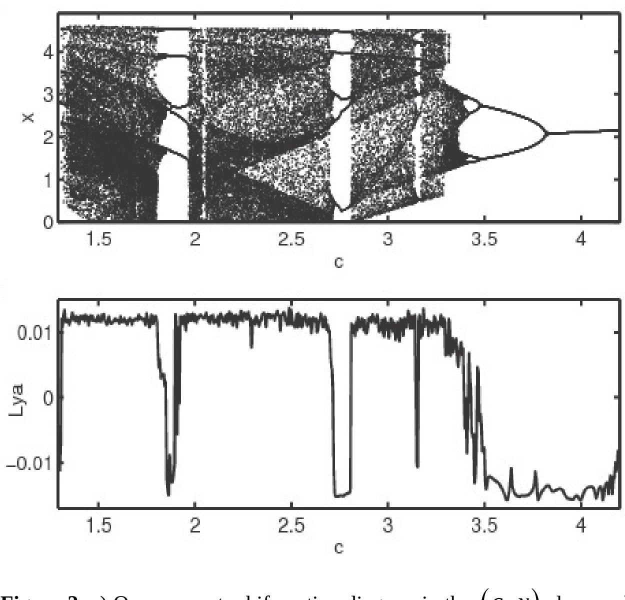

Figure 2. a) One parameter bifurcation diagram in the (c, x) plane and b) maximal Lyapunov spectrum Lya

The bifurcation diagram consists of quasiperiodicity, chaos, windows, period adding sequences and the familiar period doubling bifurcation sequence, intermittency and so on. As shown, there is a good agreement between the bifurcation diagram and its corresponding maximal Lyaponov exponent.

3. Synchronization of Coupled Oscillators

Recently, Woafo and Kraenkel [18] considered the problem of stability and duration of the synchronization process between classical Van der Pol oscillators and showed that the critical slowing-don behavior of the synchronization time and the boundaries of the synchronization domain can be estimated by analytical investigations. The next subsections extend the calculations of Ref. [18] to two, three and four diffusely coupled oscillators. Only in our analytical treatment, we assume that oscillators have identical coefficients.

3.1. Synchronization in a Case of two Oscillators

Here, we aim to determine the threshold value for synchronization (the minimal value K such that practical synchronization occurs) of two oscillators. The two oscillators are diffusely coupled with a buffer and a variable resistor R_c, which give the coupling constant $K = \left(R_c C_1 \omega_0\right)^{-1}$. The dynamics of two diffusely coupled oscillators can be described by the following set of equations:

$$\begin{cases} \dot{x}_j = y_j - z_j + K\left(x_{i\neq j} - x_j\right), \\ \dot{y}_j = ax_j - by_j - z_j - sh\left(x_j\right), \\ \dot{z}_j = x_j + dy_j, \end{cases} \tag{13}$$

where index i and j represent the oscillator number ($i, j \in \{1,2\}$). Assuming the three following vectors defined as $u_1(x_1, y_1, z_1)$, $u_2(x_2, y_2, z_2)$ and $\varepsilon(\varepsilon_1, \varepsilon_2, \varepsilon_2) = u_1 - u_2$, the stability of synchronization manifold is decided by the asymptotic behavior of $\varepsilon_1 = x_1 - x_2, \varepsilon_2 = y_1 - y_2$ and $\varepsilon_3 = z_1 - z_2$. At a linear approximation, ε_1, ε_2 and ε_3 obey to

$$\begin{cases} \dot{\varepsilon}_1 = \varepsilon_2 - \varepsilon_3 - 2K\varepsilon_1, \\ \dot{\varepsilon}_2 = \left[a - ch\left(x_1\right)\right]\varepsilon_1 - b\varepsilon_2 - c\varepsilon_3, \\ \dot{\varepsilon}_3 = \varepsilon_1 + d\varepsilon_2. \end{cases} \tag{14}$$

By replacing the chaotic variable x_1 by its virtual orbit x_0, the dynamics of the synchronization errors is then described by the linear system

$$\begin{pmatrix} \dot{\varepsilon}_1 \\ \dot{\varepsilon}_2 \\ \dot{\varepsilon}_3 \end{pmatrix} = \begin{pmatrix} -2K & 1 & -1 \\ \alpha & -b & -c \\ 1 & d & 0 \end{pmatrix} \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{pmatrix}, \tag{15}$$

with

$$\alpha = a - ch\left(x_0\right) = a - \sum_{n=0}^{N} \left[\frac{|A|^{2n}}{(n!)^2}\right].$$

Using the Lyapunov criteria, the synchronization is stable if the real part of all eigenvalues is negative. Assuming that λ is the eigenvalue of system (15), then it obeys the following algebraic third order equation:

$$\begin{aligned} &\lambda^3 + \left(b + 2K\right)\lambda^2 + \left(2Kb + cd - \alpha + 1\right)\lambda + \\ &\left(c + b + 2Kcd + \alpha d\right) = 0. \end{aligned} \tag{16}$$

The determination of signs of the real parts of the root λ may be carried out by making use of the Routh-Hurwitz criterion. In applying this criterion, we find that the real parts of the roots are negative if

$$\begin{cases} b + 2K > 0, \\ 2Kb + cd - \alpha + 1 > 0, \\ c + b + 2Kcd + \alpha d > 0, \\ 4K^2 b + \left(2b^2 - 2\alpha + 2\right)K + \left(bcd - b\alpha - \alpha d - c\right) > 0. \end{cases} \tag{17}$$

Then the synchronization is said to be stable if $K > K_{2\min}$, where the new parameter $K_{2\min}$ is defined as follow:

$$K_{2\min} = -\frac{c + b + \alpha d}{2cd}. \tag{18}$$

As shown in Figure 3, the result obtained from equation (18) is verified by a direct numerical simulation of system (13).

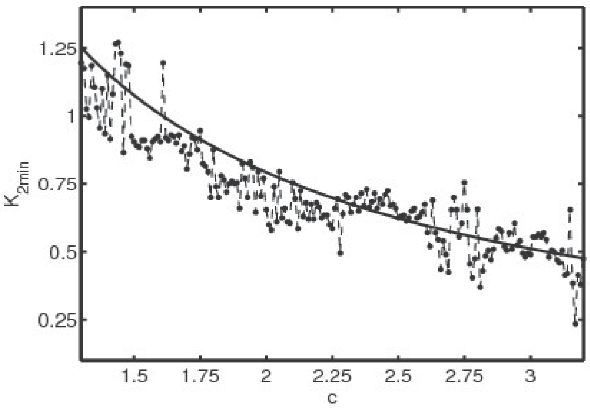

Figure 3. Synchronization boundaries in the case of two coupled oscillators

Numerically, we use the fourth order Runge Kutta algorithm and we find the first value of K for which quantity $|x_1 - x_2| < 0.05$ for $t \geq 400$. In Figure 3, the numerical result (points and dashed-line) fluctuates around the analytic curve (solid line).

3.2. Synchronization in a Case of three Oscillators

In this case, the dynamics of three diffusely coupled oscillators can be described by the following set of equations:

$$\begin{cases} \dot{x}_j = y_j - z_j + K\left(x_{j-1} - 2x_j + x_{j+1}\right), \\ \dot{y}_j = ax_j - by_j - z_j - sh\left(x_j\right), \\ \dot{z}_j = x_j + dy_j. \end{cases} \tag{19}$$

Index j (with $1 \leq j \leq 3$) represents the oscillator number and the two periodic boundary conditions $x_0 = x_3$ and $x_4 = x_1$ are used. Assuming the five following vectors defined as $u_1(x_1, y_1, z_1)$, $u_2(x_2, y_2, z_2)$, $u_3(x_3, y_3, z_3)$, $\eta(\eta_1, \eta_2, \eta_2) = u_1 - 2u_2 + u_3$, and $\varepsilon(\varepsilon_1, \varepsilon_2, \varepsilon_2) = u_1 - u_3$, the stability of synchronization manifold is decided by the asymptotic behavior of η and ε. At a linear approximation, the components of η and ε obey to

$$\begin{cases} \dot{\eta}_1 = \eta_2 - \eta_3 - 3K\eta_1, \\ \dot{\eta}_2 = \left[a - ch\left(x_1\right)\right]\eta_1 - b\eta_2 - c\eta_3, \\ \dot{\eta}_3 = \eta_1 + d\eta_2. \end{cases} \quad (20)$$

$$\begin{cases} \dot{\varepsilon}_1 = \varepsilon_2 - \varepsilon_3 - 3K\varepsilon_1, \\ \dot{\varepsilon}_2 = \left[a - ch\left(x_1\right)\right]\varepsilon_1 - b\varepsilon_2 - c\varepsilon_3, \\ \dot{\varepsilon}_3 = \varepsilon_1 + d\varepsilon_2. \end{cases} \quad (21)$$

The form of systems (20) and (21) brings the following comment: the three oscillators fall together in the synchronization. Proceeding in the same manner as the above subsection, we obtain the synchronization boundary as $K > K_{3\min}$ where

$$K_{3\min} = -\frac{c + b + \alpha d}{3cd}. \quad (22)$$

Figure 4 shows comparison between analytical result (full line) and numerical result (points and dashed-line). Numerically, we find the first value of K for which quantity $0.5\left(|x_1 - x_2| + |x_1 - x_3|\right) < 0.05$ for $t \geq 400$.

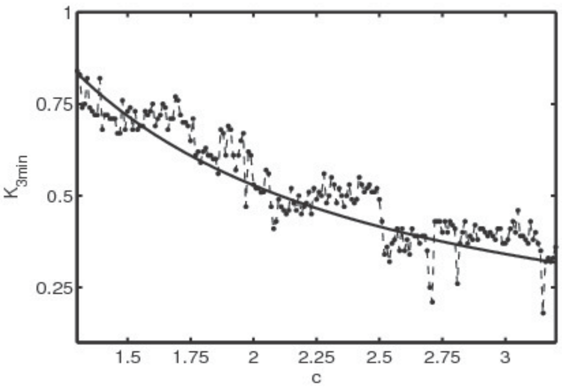

Figure 4. Synchronization boundaries in the case of three coupled oscillators

3.3. Synchronization in a Case of four Oscillators

Four systems are diffusely coupled in a ring structure with a coupling constant K. The dynamics of four diffusely coupled oscillators can be described by the following set of equations:

$$\begin{cases} \dot{x}_j = y_j - z_j + K\left(x_{j-1} - 2x_j + x_{j+1}\right), \\ \dot{y}_j = ax_j - by_j - z_j - sh\left(x_j\right), \\ \dot{z}_j = x_j + dy_j. \end{cases} \quad (23)$$

Index j (with $1 \leq j \leq 4$) represents the oscillator number and the two periodic boundary conditions $x_0 = x_4$ and $x_5 = x_1$ are used. Assuming the three following vectors defined as

$$\eta = (u_1 + u_3) - (u_2 + u_4), \ \varepsilon = u_1 - u_3 \ \text{and} \ \xi = u_2 - u_4. \quad (24)$$

The stability of synchronization manifold is decided by the asymptotic behavior of η, ε and ξ. At a linear approximation, the components of η, ε and ξ obey to

$$\dot{\eta} = M_1\eta, \quad \dot{\varepsilon} = M_2\varepsilon, \quad \dot{\xi} = M_2\xi \ \text{where}$$

$$M_1 = \begin{pmatrix} -4K & 1 & -1 \\ \alpha & -b & -c \\ 1 & d & 0 \end{pmatrix} \text{and} \ M_2 = \begin{pmatrix} -2K & 1 & -1 \\ \alpha & -b & -c \\ 1 & d & 0 \end{pmatrix}. \quad (25)$$

Choosing $K_{4min} = -\frac{c + b + \alpha d}{4cd}$, if $K_{4min} < K < 2K_{4min} = K_{2min}$ the ring falls in the cluster synchronization ($u_1 \simeq u_3$, $u_2 \simeq u_4$ while $u_1 \neq u_2$). If $K > 2K_{4min}$ the complete synchronization ($u_1 \simeq u_2 \simeq u_3 \simeq u_4$) occurs in the ring. To verify our assumption, we plot as shown in Figure 5 our analytical result ($2K_{4min}$) as function of c and our numerical result obtained while recording the first value of K for which $\frac{1}{3}\left(|x_1 - x_2| + |x_1 - x_3| + |x_1 - x_4|\right) < 0.05$ (dashed lines).

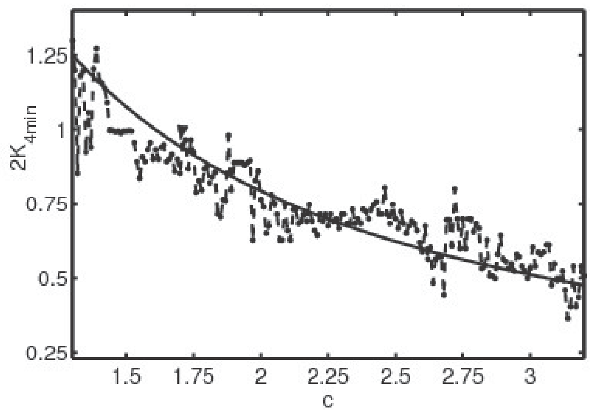

Figure 5. Synchronization boundaries in the case of four coupled oscillators

Although the cluster domain obtained analytically is verified numerically, the oscillators lost their chaotic state.

4. Experimental and Numerical Results

4.1. Experimental Setup

An experimental setup consisting of a network of four oscillators is shown in Figure 6. The networks of two and three oscillators can be derived from Figure 6 by choosing the suitable connections. In the set of equations (13), (19) and (23) the variables x_j, y_j and z_j are the voltages across the capacitors, C_{1j}, C_{2j} and C_{3j} respectively. The coupling strength between oscillators is controlled by four variable resistors R_{cj}. The circuits are built using (TL082) Operational Amplifiers, (BBY40) Diodes, Capacitances and Resistances. The nominal values of the components can be found in Table 1. Due to the tolerances of the components, oscillators are slightly different. Therefore synchronization in the sense that $|u_i(t)-u_j(t)|=0$ is not possible and practical synchronization is defined as $|u_i(t)-u_j(t)|\le\delta$ with $\delta\ll 1$.

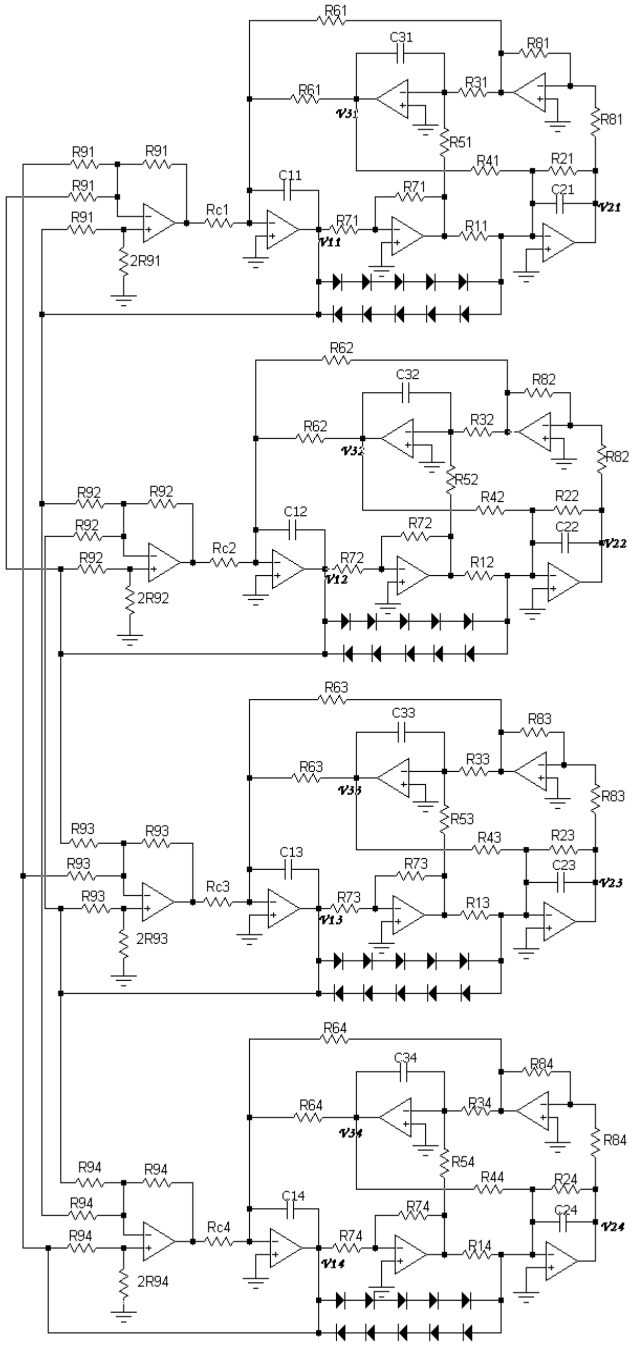

Figure 6. Electronic schematic of overall system

4.2 Experimental and numerical results

Before the coupling, Figure 7(a) shows a phase portrait of the first oscillator, which corresponds to a chaotic phase portrait (Figure 7(b)) obtained numerically with the following normalized parameters: $a_1=3.522$, $b_1=0.513$, $c_1=2.511$ and $d_1=1.264$.

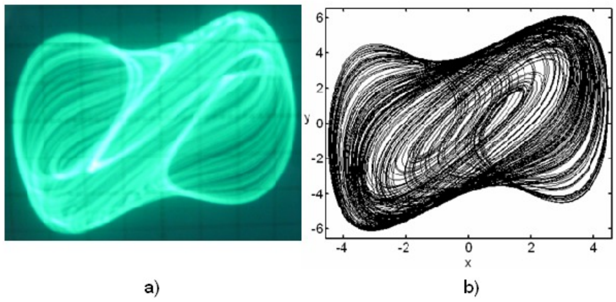

Figure 7. Phase portrait of one oscillator before the coupling. a) Experimentally obtained, b) Numerically plot

Maintaining the same parameters used in Figure 7, we show in Figure 8 the phase portraits (x_1, x_2) to illustrate the absence of synchronization in the system before the coupling.

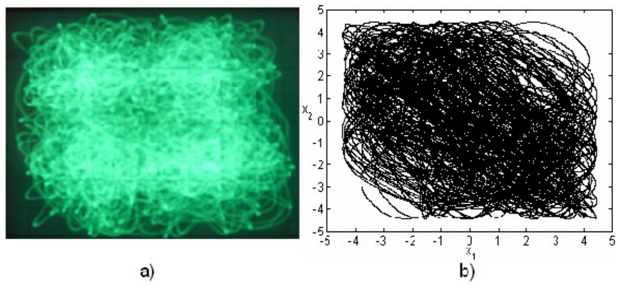

Figure 8. Phase portrait in the plane (x_1, x_2) before the coupling. a) Experimental result, b) Numerical result

After setting the coupling, we decrease the values of the resistances R_{cj} to find experimentally the synchronization domain.

Table 1. Values of capacitors and resistances used in the overall electronic circuit

Resistances (Ω) and Capacitances (nF)	First Oscillator $(j=1)$	Second Oscillator $(j=2)$	Third Oscillator $(j=3)$	Fourth Oscillator $(j=4)$
C_{1j}	12.0	12.5	12.6	12.3
C_{2j}	192.8	201.2	202.6	198.0
C_{3j}	10.1	10.2	10.2	10.2
R_{1j}	177.4	170.2	168.8	173.1
R_{2j}	1218	1158	1154	1170
R_{3j}	9437	9329	9366	9337
R_{4j}	248.9	238.7	238.7	242.3
R_{5j}	11920	11810	11810	11820
R_{6j}	10040	10060	10050	10060
R_{7j}	35100	35140	35170	35110
R_{8j}, R_{9j}	$18\cdot10^4$	$18\cdot10^4$	$18\cdot10^4$	$18\cdot10^4$

4.2.1. Two Oscillators

In this case, we find that for $R_{cj}\le 400\Omega$, $(j=1,2)$, the system falls in the synchronization. Figure 9(a) and 9(b) illustrate the complete synchronization state of the ring for $R_{c1} = 315.0\Omega$, $R_{c2} = 318.7\Omega$ and $K_1 = 0.813$, $K_2 = 0.815$. Another used parameters are the following: $a_1 = 3.522, b_1 = 0.513, c_1 = 2.511, d_1 = 1.264$, $a_2 = 3.518$, $b_2 = 0.517, c_2 = 2.508$ and $d_2 = 1.266$.

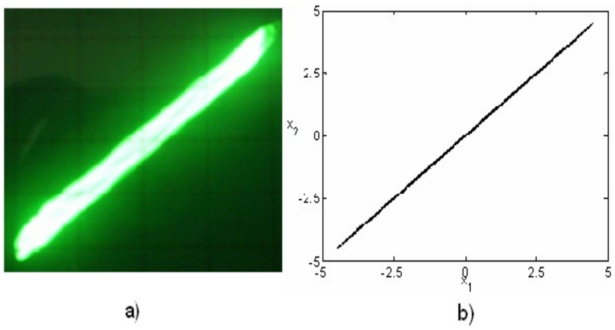

Figure 9. Phase portraits in the plane (x_1 , x_2) illustrating synchronization in the ring of two coupled oscillators. a) Experimental result, b) Numerically plot

4.2.2. Three oscillators

In the case of three coupled oscillators, when the coupling resistances satisfy $R_{cj} \le 298\Omega$, $(j = 1, 2, 3)$, the ring is in the complete synchronization. To prove it, we plot in Figure 10 some phase portraits: Figure 10(a) (resp. Figure 10(c)) represents our experimental result in the plane (x_1, x_2) (resp. (x_1, x_3)) while Figure 10(b) (resp. Figure 10(d)) is its numerical analogous. The coupling resistances are $R_{c1} = 237.2\Omega$, $R_{c2} = 240.5\Omega$, $R_{c3} = 237.8\Omega$ and $K_1 = 0.53$, $K_2 = 0.52$, $K_3 = 0.55$. Other parameters are keep constant: $a_1 = 3.522, b_1 = 0.513,$ $c_1 = 2.511,$ $d_1 = 1.264$, $a_2 = 3.518,$ $b_2 = 0.517,$ $c_2 = 2.508$, $d_2 = 1.266$, $a_3 = 3.525,$ $b_3 = 0.515,$ $c_3 = 2.513$ and $d_3 = 1.261$.

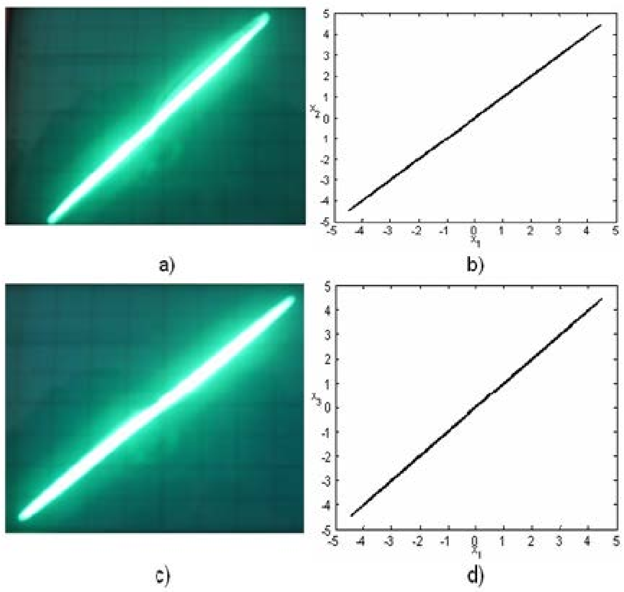

Figure 10. Phase portraits illustrating the complete synchronization in the ring of three coupled oscillators. a) Experimental result in the plane (x_1, x_2), b) Numerically plot in the plane (x_1, x_2), c) Experimental result in the plane (x_1, x_3), d) Numerically plot in the plane (x_1, x_3)

4.2.3. Four Oscillators

Although the analytical study foresaw a cluster synchronization, the numerical and the experimental studies showed only a complete synchronization state. This is obtain experimentally when $R_{cj} \le 345\Omega$, $(j = 1, 2, 3, 4)$. Proceeding as the above subsection, we plot in Figure 11 different phase portraits: Figure 11(a), Figure 11(c) and Figure 11(e) represent our experimental results drawn in the planes (x_1, x_2), (x_1, x_3) and (x_1, x_4) respectively, while Figure 11 (b), Figure 11 (d) and Figure 11 (f) are their numerical corresponding. These following coupling parameters are used: $R_{c1} = 327.2\Omega$, $R_{c1} = 325.5\Omega$, $R_{c1} = 328.4\Omega$, $R_{c1} = 324.8\Omega$ and $K_1 = 8.03$, $K_2 = 8.025$, $K_3 = 8.027$, $K_4 = 8.031$, $a_1 = 3.522,$ $b_1 = 0.513,$ $c_1 = 2.511,$ $d_1 = 1.264$, $a_2 = 3.518,$ $b_2 = 0.517,$ $c_2 = 2.508$, $d_2 = 1.266$, $a_3 = 3.525, b_3 = 0.515, c_3 = 2.513$, $d_3 = 1.261$, $a_4 = 3.515,$ $b_4 = 0.520,$ $c_4 = 2.511$ and $d_4 = 1.265$.

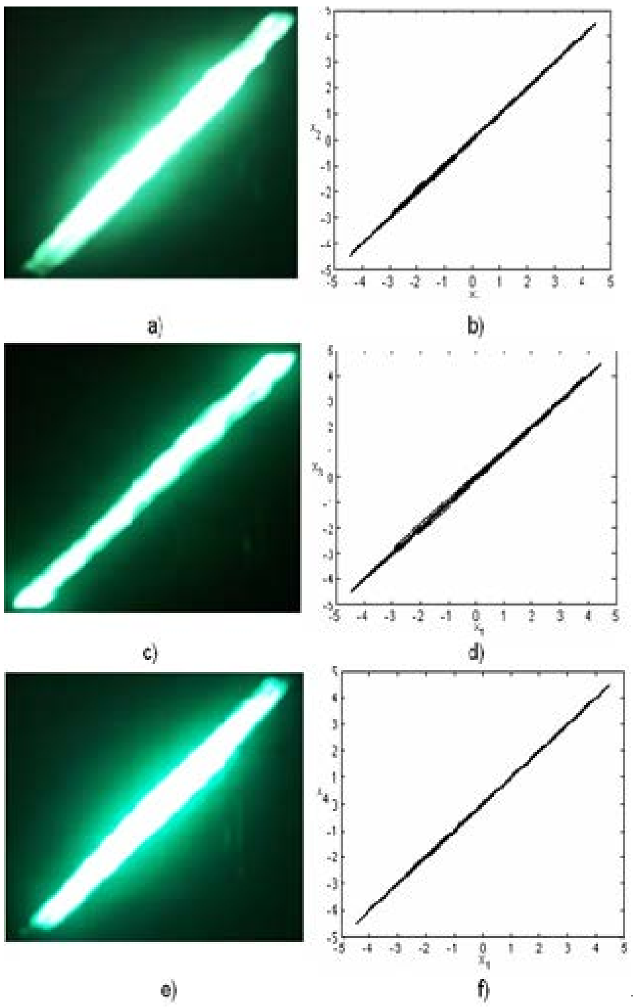

Figure 11. Phase portraits illustrating the complete synchronization in the ring of four coupled oscillators. a) Experimental result in the plane (x_1 , x_2), b) Numerically obtained in the plane (x_1 , x_2), c) Experimental result in the plane (x_1, x_3), d) Numerically obtained in the plane (x_1, x_3), e) Experimental result in the plane (x_1, x_4), f) Numerically obtained in the plane (x_1, x_4)

5. Conclusion

In this paper, theoretical and experimental complete synchronization of diffusively two, three and four coupled oscillators are presented. With the experimental setup it is impossible to achieve a zero synchronization error due to the tolerances of the electrical components. We obtain that three and four diffusely coupled oscillators synchronized or desynchronized together, provided initial values are chosen in the vicinity of the synchronization manifold. Despite the fact that the single harmonic response give in equation (9) may be questionable, the analytical treatment gives a good indication on the boundary of K for synchronization to be achieved. The presented experimental results are qualitative comparable with numerical simulations.

Acknowledgement

This work is supported by the Academy of Sciences for the Developing World (TWAS) under Research grant N.03-322 RG/PHYS/AF/AC.

References

[1] Ling, L. and Le, M, "Parameter identification and synchronization of spatiotemporal chaos in uncertain complex network," Nonlinear Dynamics 66 (4). 489-495. Dec. 2011.

[2] Carroll, T.L. and Pecora, L.M, "Synchronizing chaotic circuits," IEEE Trans Circ Syst 38 (2). 453-456. Apr. 1991.

[3] Yu P.E. and Kuznetsov A.P, "Synchronization of coupled van der pole and Kislov-Dmitriev self-oscillators," Technical Physics 56, (4). 435-442. Apr. 2011.

[4] Peng J.H., Ding, E.J., Ding, M. and Yang, W, "Synchronizing hyperchaos with a scalar transmitted signal," Phys Rev Lett 76 (6). 904-907. 1996.

[5] Nijmeijer H, "Control of chaos and synchronization," Systems and Control Letters 31 (5). 259-262. Oct. 1997.

[6] Kennedy M.P., Rovatti R. and Setti, G, "Chaotic Electronics in Telecommunications," CRC Press, USA. 2000.

[7] Gamal M.M., Shaban A.A. and AL-Kashif M.A, "Dynamical properties and chaos synchronization of a new chaotic complex nonlinear system," Nonlinear Dynamics. 51, 171-181 Oct. 2008.

[8] Tanmoy B., Debabrata B. and Sarkar B.C, "Complete and generalized synchronization of chaos and hyperchaos in a coupled first-order time-delayed system," Nonlinear Dynamics. 71 (1). 279-290. Jan. 2013.

[9] Buric N. and Vasovic N, "Global stability of synchronization between delay-differential systems with generalized diffusive," Chaos Solitons and Fractals. 31 (2). 336-342. 2007.

[10] Fotsin H. and Bowong S, "Adaptive control and synchronization of chaotic systems consisting of Van der Pol oscillators coupled to linear oscillators," Chaos Solitons and Fractals. 27 3). 822-835. Apr. 2006.

[11] Carroll T.L. and Pecora L.M, "Nonlinear Dynamics in Circuits," World Scientific Publishing, Singapore. 1995.

[12] Juan C., Jun-an L. and Xiaoqun W, "Bidirectionally coupled synchronization of the generalized Lorenz systems," Journal of Systems Science and Complexity 24 (3). 433-448. Jun. 2011.

[13] Pecora L.M. and Carroll T.L, "Driving systems with chaotic signals," Physl Rev A. 44 (4). 2374-2383. Aug. 1991.

[14] Cuomo K.M. and Oppenheim A.V, "Circuit implementation of synchronized chaoswith application to communications," Phys Rev Lett. 71 (1). 65-68. Jul. 1993.

[15] Nana B. and Woafo P, "Synchronization in a ring of four mutually coupled Van der pol oscillator: Theory and experiment," Phys Rev E 74, 1-8. 2006.

[16] Yu Y.H., Kwak K., Lim T.K, "Synchronization via small continuous feedback," Phys Lett A. 191 (3). 233-237. 1994.

[17] Li-feng Z., Xin-lei A. and Jian-gang Z, "A new chaos synchronization scheme and its application to secure communications," Nonlinear Dynamics. 73 (2). 705-722. Jan. 2013.

[18] Woafo P. and Kraenkel R.K, "Synchronization stability and duration time," Phys Rev E. 66. 25-31. 2002.