

# Design and Analysis of Losses in Power Transformer

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**Abstract** This paper examined and presented a simplified approach to power transformer design. Analyzed possible losses associated with power transformers through computational techniques and crucial design diagram.

**Keywords:** core design, window area, window space factor, cruciform design, winding arrangement, transformer loss analysis

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## 1. Introduction

Transformer is a static electrical device that transfers energy by inductive coupling between its winding circuits [1]. A varying current in the primary windings creates a varying magnetic flux through the secondary winding by which power is delivered to the load. Power transformer is slightly different from potential and current transformers in the sense that it transforms both current and voltage at the same time though they all work using the same principle of electromagnetic induction.

## 2. Construction Details of Transformer

Single phase shell type transformer is represented in the sketch below. Core type transformer exists, in which the primary and the secondary windings are located on the different limb. As the iron area of the leg  $A_i$  and the window area  $A_w = (\text{Height of window } L * \text{Window width})$  increases, the size of the transformer also increases. In general the size of the transformer increases as the output of the transformer increases [2],

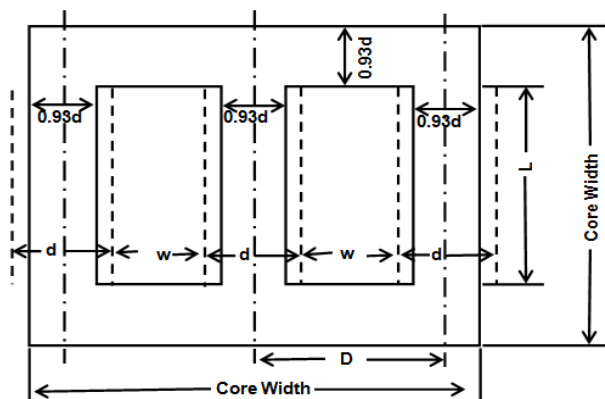


Figure 1. Single phase shell type transformer

- NB. W = Width of the transformer
- H = Height of the transformer.
- $W_w$  =  $W_s$  = Window space width
- $W_c$  = Window space when insulation thickness is removed.
- D = Thickness of the core
- L = Height of the window
- $L_c$  = Window height when insulator is subtracted.

## 3. Winding Arrangement

The low voltage winding is placed next to the core and high voltage winding over the low voltage winding, on the central limb, in order to reduce the quantity of insulation used, avoid possibility of breakdown of space between the core and high voltage coil [3,4], and to control the leakage reactance. However, in the case of low voltage rated transformers, low and high voltage coil can be arranged in any manner, as in this design, but to obtain optimum performance, we arranged the coils as shown below.

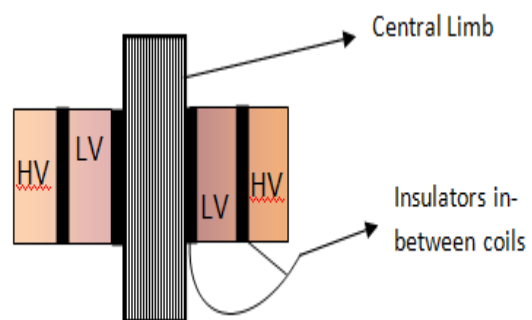


Figure 2. Winding arrangement

## 4. Core Design

Net iron Area of the leg or limb or Core

$$A_i = \Phi / B_m \quad (1)$$

The effect of mechanical forces on the wound coil under short circuit or high overload condition tends to deform the geometry of the transformer or its structure. These forces destroys the coil and insulations, in rectangular or square wound coil and core, whereas the effect is negligible in the case of circular wound coil and core or cruciform core design [5,6]. The circular core calls for more number of different sizes of laminations and poses a problem during core anchoring. Hence cruciform core design is preferred in practical design of transformers. Cruciform core design, reduces the net area of the core, gives high space factor, reduces the mean length of turns, and consequent I<sup>2</sup>R loss, as demonstrated in the two core design comparison below.

### 5. Leg or Limb Section Details

#### 1. Square core (With a circular coil)

$$a = \text{width of the stamping or leg} \tag{2}$$

$$= d \cdot \sin 45^\circ \text{ or } d \cdot \cos 45^\circ = 0.71 \cdot d$$

where d is the diameter of the circumscribing circle.

$$\text{Iron area } A_i = k_i \cdot a^2 = K_i \cdot (0.71 \cdot d)^2 \tag{3}$$

= 0.9 \* 0.5 \* d<sup>2</sup> for 10% insulation or K<sub>i</sub> = 0.9 (Iron or staking factor)

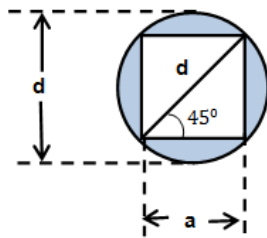


Figure 3. Square cores with circular coil

Therefore Iron area;

$$A_i = 0.45 \cdot d^2. \tag{4}$$

Area of circumscribing circle

$$A_{cc} = \pi \cdot d^2 / 4 = 3.14 \cdot d^2 / 4 = 0.785 \cdot d^2. \tag{5}$$

Therefore

$$A_i / A_c = 0.45 \cdot d^2 / 0.785 d^2 \tag{6}$$

$$= 0.573 \text{ (window space factor } K_w)$$

NB: It is clear that A<sub>i</sub> is only 57.3% of A<sub>c</sub> Rest of the area 42.7% of A<sub>c</sub> is not being utilized usefully. In other to utilize the area usefully, more number of stepping of the core is necessary as shown below.

#### 2. Cruciform or 2 stepped core and circular coil.

a = width of the largest stamping  
 b = width of the smallest stamping  
 Gross area of the core

$$A_g = 2 \cdot a \cdot b - b^2 \tag{7}$$

Since a = d \* Cosθ, and b = d \* Sinθ -from the figure above.

Substituting in equation (7)

$$A_g = 2 \cdot a \cdot b - b^2 = 2 \cdot d^2 \cdot \cos\theta \sin\theta - d^2 \cdot \sin^2\theta. \tag{8}$$

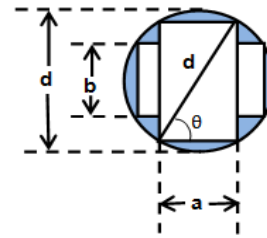


Figure 4. Cruciform cores with circular coil

By trigonometric identities,

$$\cos\theta \sin\theta = \sin 2\theta \tag{9}$$

Therefore,

$$A_g = 2 \cdot d^2 \cdot \sin 2\theta - d^2 \cdot \sin^2\theta. \tag{10}$$

For A<sub>g</sub> to be maximum, A<sub>g</sub> must be differentiated with respect to θ, A<sub>g</sub> = d<sup>2</sup> \* (Sin2θ - Sin<sup>2</sup>θ)

Hence;

$$dA_g / d\theta = d^2 (2\cos 2\theta - \sin\theta \cos\theta) \tag{11}$$

$$\text{i.e } 2\cos 2\theta - \sin 2\theta = 0$$

$$= 2\cos 2\theta = \sin 2\theta.$$

$$2 = \sin 2\theta / \cos 2\theta = \tan 2\theta, \text{ Therefore,}$$

$$\tan 2\theta = 2, 2\theta = \tan^{-1} 2 \tag{12}$$

$$\theta = \tan^{-1} 2 / 2 = 63.43 / 2 = 31.71^\circ.$$

Thus, A<sub>g</sub> is maximum when θ = 31.71°. With θ = 31.71°

$$a = d \cdot \cos 31.71^\circ = 0.85 \cdot d, \tag{13}$$

$$b = d \cdot \sin 31.71^\circ = 0.53 \cdot d.$$

$$A_g = 2 \cdot a \cdot b - b^2 \tag{14}$$

$$= 2 \cdot 0.85 \cdot d \cdot 0.53 \cdot d - (0.53d)^2 = 0.62d^2.$$

$$A_i = k_i A_g = 0.9 \cdot 0.62d^2 = 0.56d^2. \tag{15}$$

$$\text{And } A_i / A_c = 0.56d^2 / 0.785d^2 \tag{16}$$

$$= 0.71 = 71\% \text{ - better window utilization.}$$

### 6. Mean Length of Turns

For a given Area A<sub>i</sub> say 10cm<sup>2</sup>,

(a) Square core (with a circular coil)

A<sub>i</sub> = 0.45 \* d<sup>2</sup> as obtained in the previous analysis of equation (3).

$$\text{Circumference} = \pi \cdot d - \text{mean length of turns} \tag{17}$$

$$10 = 0.45 \cdot d^2, 10 / 0.45 = d^2.$$

$$\text{Therefore: } d = \sqrt{22.22} = 4.71$$

Hence,

$$\text{mean length of turns } \pi * d = 3.14 * 4.71 = \mathbf{14.80cm.} \quad (18)$$

(b) Cruciform-2 stepped core with circular coil.

$$\text{Circumference} = \pi * d - \text{mean length of turns}$$

$$10 = 0.56 * d^2, 10 / 0.56 = d^2.$$

$$\text{Therefore: } d = \sqrt{17.857} = 4.226$$

$$\begin{aligned} \text{Hence mean length of turns } \pi * d \\ = 3.14 * 4.226 = \mathbf{13.28cm.} \end{aligned} \quad (19)$$

Therefore, the addition of one step to the square core enhanced the utilization of more space of the circumscribing circle and reduced the mean length of turns by a difference of 1.52cm for the same number of turns. This result in more copper savings; i.e better economy of design.

## 7. Yoke Section Details.

The purpose of the yoke is to connect the legs providing a least reluctance path. In other to limit iron loss in the yoke, operating flux density is reduced by increasing yoke area. Generally, yoke area is made 20% more than the leg areas. i.e.

$$0.2 * A_l = A_y \quad (20)$$

Where  $A_y$  – yoke area  
 $A_l$  – core or leg area.

## 8. Window Area and Core Proportion

$$\begin{aligned} \text{Area of the window } A_w \\ = \text{KVA} / (2.22 * f * A_l * B_m * J * K_w * 10^{-3}) \text{ (meter}^2 \text{)}. \end{aligned} \quad (21)$$

If  $H_w$  = height of the window,  $W_w$  = width of the window, then  $A_w = H_w * W_w$ . In other to limit the leakage reactance of the transformer,  $H_w$  is made more than  $W_w$ . In practice,  $H_w/W_w$  lies between 2.5 and 3.5.

## 9. Winding Details

Since the applied voltage  $V_1$  is approximately equal to the voltage induced

$$E_1 = 4.44 * \Phi_m * f * N_1 = E_t N_1. \quad (22)$$

$$\begin{aligned} \text{Number of primary turns (or turns / phase) } N_1 \\ = V_1 / E_t - \text{Single phase transformer.} \end{aligned} \quad (23)$$

$$\begin{aligned} \text{Number of Secondary Turns (or turns / phase) } N_2 \\ = V_1 / E_t - \text{Single phase transformer.} \end{aligned} \quad (24)$$

$$\begin{aligned} \text{Primary Current (or current / phase) } I_1 \\ = \text{KVA} * 10^3 / V_1 - \text{Single phase transformer.} \end{aligned} \quad (25)$$

Cross sectional Area of primary winding conductor

$$a_1 = I_1 / J \text{ (mm}^2 \text{)} \quad (26)$$

Secondary current (or current/phase)

$$I_2 = \text{KVA} * 10^3 / V_2 - \text{Single phase transformer.} \quad (27)$$

Cross sectional area of secondary winding conductor

$$a_2 = I_2 / J \text{ (mm}^2 \text{)}. \quad (28)$$

## 9. Output Equations

The output equation for a Single phase shell type transformer is stated as follows:

$$VA = V_1 I_1 = E_1 I_1 = 4.44 \Phi_m f N_1 I_1 \quad (29)$$

Where  $\Phi_m$  = Maximum flux,  $f$  = frequency of operation

$N_1$  = Number of coil turns in the primary coil.

$I_1$  = Electrical current in primary coil, which sets up the magnetizing flux.

4.44 – constant obtained from form factor.

## 10. Change of Flux in the Core

Since the flux is assumed to be sinusoidal, the maximum flux  $\Phi_m$  is achieved at quarter the period of oscillation.

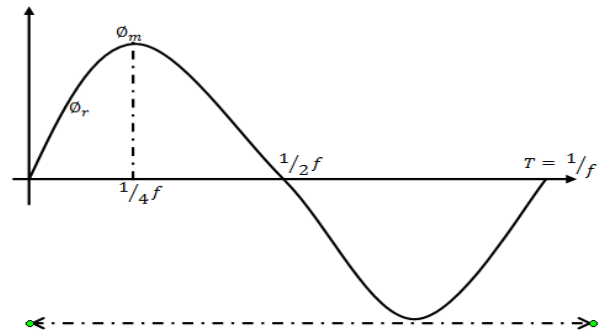


Figure 5. Wave form of a sinusoidal flux

Hence, average rate of change of flux

$$\Delta \Phi = \Phi_m / (T/4) = \Phi_m * 4 / T = 4 \Phi_m / T \text{ Wb/s or volt.} \quad (30)$$

But  $T = 1/f$ ; this implies

$$4 \Phi_m / (1/f) = 4 \Phi_m * f = 4f \Phi_m \text{ Volts.} \quad (31)$$

Form factor = r.m.s value/average value = 1.11, Therefore

$$\text{r.m.s value of e.m.f / turn} = 1.11 * 4 * f * \Phi_m = 4.44f \Phi_m \quad (32)$$

Comparing with equation (29),

Power delivered  $VA = V_1 I_1 = E_1 I_1 = 4.44f \Phi_m N_1 I_1$  or Power delivered per turn,

$$VA / N = V_1 I_1 / N_1 = E_1 I_1 / N_1 = 4.44f \Phi_m I_1. \quad (33)$$

But  $\Phi_m = B_m * A_i$ , where  $B_m$  – maximum flux density,  $A_i$  = iron area or core area.

In a shell type transfer, there are two windows, which are symmetrical to each other. Hence it is sufficient to design one side. Since the low and high voltage windings are placed on the central limb, each window accommodates  $T_1$  and  $T_2$  turns of both primary and secondary windings.

Area of copper in the window

$$A_{cu} = a_1 N_1 + a_2 N_2 = (I_1 N_1 / J) + (I_2 N_2 / J) \quad (34)$$

$$= 2I_1 N_1 / J = A_w K_w$$

$$\rightarrow I_1 N_1 = I_2 N_2 = A_w K_w J / 2. \quad (35)$$

Substituting in equation (29) above gives,

$$\text{KVA} = 4.44 * \Phi_m * f * N_1 I_1 * 10^{-3}. \text{ Also } \Phi_m$$

$$= A_i B_m \text{KVA} = (4.44 A_i B_m f A_w K_w J) / 2 \quad (36)$$

$$= 2.22 * A_i * B_m * f * A_w * K_w * J * 10^{-3}.$$

$$\text{VA} = 2.22 * A_i * B_m * f * A_w * K_w * J \quad (37)$$

- the output equation of a transformer.

### 11. Window Space factor

Window space factor is defined as the ratio of copper area in the window to the area of the window.

That is  $K_w = \text{Area of copper in the window } A_{cu} / \text{Area of the window } A_w$ .

For a given window area, as the voltage rating of the transformer increases, quantity and quality of insulation in the window increases [7, 8], area of copper reduces. Thus the window space factor reduces as the voltage increases.

A value for  $K_w$  can be calculated by the following empirical formula

$$K_w = 10 / (30 + KV_{hv}) \quad (38)$$

Where  $KV_{hv}$  or  $V_{hv}$  is the voltage of the high voltage winding expressed in KV or V.

### 12. Transformer Loss Analysis

Transformer losses fall into three categories:

1. No-load loss, or iron loss
2. Load-loss, copper loss, or short-circuit loss
3. Stray-loss (that is largely load related).

For some larger transformers there are also losses due to fans and pumps providing forced cooling. This will be discussed in a later example [9].

### 13. No Load Current of a Transformer

The no-load current  $I_0$  is the vectorial sum of the magnetizing current  $I_m$  and core loss or working component current  $I_c$ . Function of  $I_m$  is to produce flux  $\Phi_m$  in the magnetic circuit and the function of  $I_c$  is to satisfy the no load losses of the transformer. Thus:  $I_0 = \sqrt{(I_c^2 + I_m^2)}$ , Ampere.

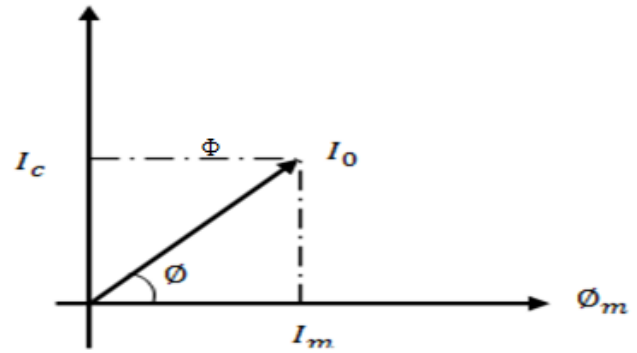


Figure 6. Vector diagram of transformer under no-load condition

No load input to the transformer =  $V_1 I_0 \cos \Phi_0 = V_1 I_c =$  No load losses as the output is zero and input = output + losses.

Since, the copper loss under no load condition is almost negligible, the no load losses can entirely be taken as due to core loss only. Thus the core loss component of the no load current  $I_c = \text{Core loss} / V_1$  for single phase transformer as in this case.

### 14. Transformer Approximation at no-load

1. In case of a transformer of normal design, the no load current will generally be less than about 2% of the full load current.
2. No load power factor  $\cos \Phi_0 = I_c / I_0$  and will be around 0.2.
3. Transformer copper loss.
  - (a) The primary copper loss, at no load is negligible as  $I_0$  is very less.
  - (b) The secondary copper loss is zero at no load, as no current flows in the secondary winding.
4. **Core or Iron loss.** Total core loss = loss in leg and yokes. The core loss can be estimated at design stage by referring to graph of core loss/kg versus flux density.

Core loss in leg = loss/kg in leg \* weight of leg in kg = loss/kg in leg. = loss/kg in leg \* volume of the leg ( $A_l H_w$ ) \* density of steel or iron used.

Core loss in yoke = loss/kg in yoke \* volume of yoke ( $A_y$  \* mean length of the yoke \* density of iron used.)

NB: The density of iron or steel used for the transformer core design lies between 7.55-7.8 grams/cm<sup>2</sup>

### 15. Transformer on Load

The current  $I_2$  setup in the transformer when loaded is in phase with  $V_2$  if the load is resistive, lags behind  $V_2$  if the load is inductive, and leads it if the load is capacitive.

The presence of  $I_2$  in the secondary sets up demagnetizing ampere-turn  $\Phi_2$ , which reduces magnetizing ampere-turn  $\Phi$  and  $V_1$  becomes higher than  $E_1$ . The excesses of  $V_1$  causes more current  $I_2$  known as "load component of primary current" to flow, thereby setting up its own mmf  $\Phi_2$ . The interaction of this

additional mmf cancels the effect of the demagnetizing mmf  $\Phi_2$ , previously set-up by  $I_2$  in the transformer.

Hence, whatever the load condition, the net flux passing through the core is approximately the same as at no-load. A vital deduction is that due to the constancy of core flux at all load; the “core loss” is also practically the same under all load conditions.

As  $\Phi_2 = \dot{\Phi}_2$  Therefore,

$$N_2 I_2 = N_1 \dot{I}_2, \text{Hence } \dot{I}_2 = N_2 / N_1 * I_2 = k I_2.$$

**Equivalent Resistance/Reluctance /Inductance of a Transformer**

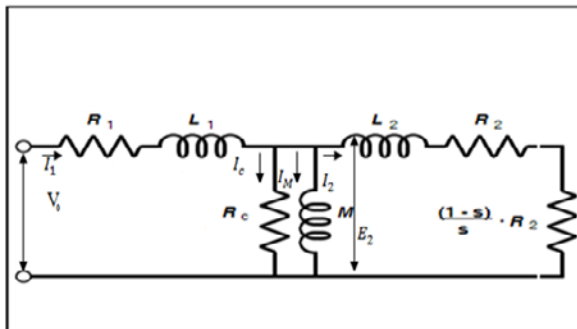


Figure 7. Equivalent resistance of a transformer

The  $R_2'$  – the equivalent secondary resistance as referred to primary. Copper loss in secondary is  $I_2^2 R_2$ , which is supplied by primary which takes a current of  $I_1$ .

Hence if  $R_2'$  is the equivalent resistance in primary which would have caused the same loss as  $R_2$  in secondary, then

$$I_1^2 R_2' = I_2^2 R_2 \text{ or } R_2' = (I_2 / I_1)^2 R_2. \tag{39}$$

Neglecting no-load current  $I_0$ , then  $I_2/I_1 = 1/k$ , Where  $R_2' = R_2/k^2$ . Referring  $R_1$  = to secondary, that is equivalent primary resistance referred to secondary, we have;

$$I_2^2 R_1' = I_2^2 R_1. \text{Therefore } R_1' = (I_1 / I_2)^2 R_1 = k^2 R_1. \tag{40}$$

**16. Stray Loss**

**Winding Eddy Current Loss**

Eddy-current loss is a significant additional component of winding loss. Winding eddy-currents are produced as a result of the alternating leakage flux cutting the windings. This flow within the conductor’s perpendicular to the load current path. The eddy-current losses are a fixed proportion of the load-losses for a particular winding. They do however vary as the square of the frequency, so that the presence of any harmonics in the load current leads to significant additional eddy-current loss [10].

For decades, eddy-current losses have presented an obstacle to reduction of  $I^2R$  losses within transformer windings. This is because increasing the conductor cross-section with the objective of reducing winding resistance had the effect of worsening the eddy-current component, and little overall benefit was obtained. Since the mid-1960s, continuously transposed conductors which

consist of a large number of individually enamelled insulated strands to increase the resistance of the eddy-current paths have been available and have largely eliminated this problem (Figure 8). Its use, coupled with flux shunts to control the distribution of leakage flux [11], means that eddy-current losses can now normally be contained within 10% to 15% of the  $I^2R$  loss. Therefore, the reduction of load loss depends simply on the amount of materials copper and iron that is considered economical to put into the transformer.



Figure 8. Eddy-currents in a solid conductor and in continuously transposed conductors.

**Mechanical Metalwork Eddy Current Loss**

These losses are those that occur in leads and tanks and other structural metalwork. Until the recent development of computer calculation techniques using finite element analysis, the magnitude of stray losses was usually determined empirically. The tolerances on guarantees took care of instances where designs did not quite conform to previous experience. Modern computer programmes have removed the uncertainty from this aspect of design and have facilitated improvements in the designs themselves [12]. They enable designers to calculate and compare losses for differing arrangements as well as enabling suitable flux shields to be placed in critical areas. Stray loss, which is load dependent, has thus been reduced from perhaps 10% of the load losses to approximately half this value.

**17. Sample Design**

**Specifications:**

1. Output KVA S = 2.22\*f\*B<sub>m</sub>\*A<sub>i</sub>\*J\*A<sub>w</sub>\*K<sub>w</sub> = 450W.
2. Voltage ratio V<sub>1</sub>/V<sub>2</sub> 220/18V
3. Frequency f. H<sub>z</sub> 50H<sub>z</sub>
4. Number of phase Single phase
5. Rating Continuous
6. Cooling Natural
7. Type Shell type

**Design Calculations**

Applying voltage transformation ratio  $E_2 / E_1$  (41)  
 $= N_2 / N_1 = k$  we have,  $18 / 220 = 0.082 = k$ .

Employing a two step cruciform core because of its gain,

$$A_i = k_i A_g \tag{42}$$

Where  $k_i$  = stacking or iron factor  
 $A_g$  = gross area of the core.

Assuming stacking factor of 0.9

$A_i = 0.9 * 0.62d^2$ , where  $d$  = diameter of the circumscribing circle or coil inner diameter or core diameter.



Assuming  $d = 5\text{cm}$  – by measurement.

Then

$$A_i = 0.56d^2 = 0.56 * 25 = 14\text{cm}^2 = \mathbf{0.14\text{m}^2}. \quad (43)$$

Applying emf equation

$$E_1 = 4.44fN_1B_m A_i N_i \quad (44)$$

$$= 220 / (4.44 * 50 * 0.42 * 0.14) = 220 / 13.05 = 16.86.$$

**18 turns – next even integer.**

Using voltage transformation ratio

$$E_2 / E_1 = N_2 / N_1 = k,$$

$$N_2 = kN_1 = 0.082 * 18 = 1.476 \text{ or } 1.48 \quad (45)$$

$$= \mathbf{2\text{turns.} - \text{Next even integer.}}$$

Voltage per turn.- should be the same for both windings in an ideal design.

Therefore;

$$\text{Emf}_1 / N_1 = 220 / 16.86 = 13.048\text{V} = \mathbf{13\text{V}}. \quad (46)$$

$\text{Emf}_2 / N_2 = 18 / 1.48 = 12.162 = \mathbf{12\text{V}} \gg \mathbf{1\text{v}}$  difference accounts for stray loss (design loss)

Maximum flux  $\Phi_m$

$$A_i = \mathbf{0.14\text{m}^2}.$$

Maximum flux through the core

$$\Phi_m = B_m * A_i = 0.42 * 0.14 = \mathbf{0.059\text{ Weber}}. \quad (47)$$

$$A_y = 0.2 * A_i = 0.2 * 0.14$$

$$= \mathbf{0.028\text{m}^2} - \text{Yoke area is increased by } 0.2 \text{ to limit } (48)$$

operating flux density at the yoke.

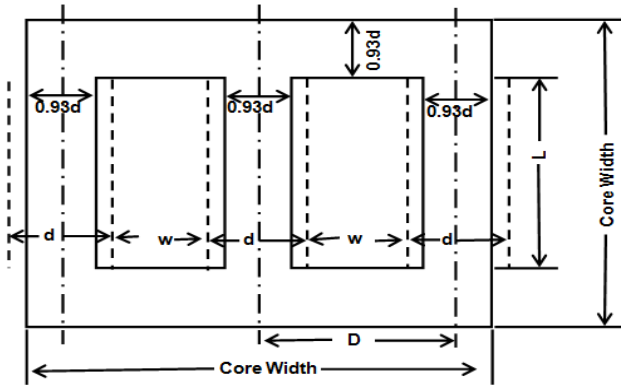


Figure 9. Single phase shell type transformer [12]

From the diagram above, overall length or width can be obtained as follows:

$$W = d + W_w + 2d + W_w + d = 2W_w + 4d - \text{over all width} \quad (49)$$

$H = L + 2L_y$  where  $L_y = \text{yoke height}$ . Assuming  $L_y = d$  from the diagram, then;

$$H = L + 2d - \text{over all height} \quad (50)$$

$$H * W = (2W_w + 4d)(L + 2d) - \text{entire area} \quad (51)$$

$$= 2W_w L + 4W_w d + 4dL + 8d^2. \quad (52)$$

Area of yoke and core =  $\mathbf{0.168\text{m}^2}$

Now, window area =  $2W_w L$ ,

Therefore, Area of yoke and core = entire area (minus) window area.

That is;

$$2W_w L + 4W_w d + 4dL + 8d^2 - 2W_w L - \text{simplifying, we have} \quad (53)$$

$$8d^2 + 4W_w d + 4dL = 4d(2d + W_w + L)$$

NB:  $H/W_w = 2.5$ , therefore  $H = 2.5W_w = L$ ,  $d = 5\text{cm} = \mathbf{0.05\text{m}}$  – as obtained from measurement as mentioned previously.

Substituting these values in the expression above we have,

$$4 * 0.05(2 * 0.05 + W_w + 2.5W_w) = 0.168$$

$$0.2(0.1 + W_w + 2.5W_w) = 0.168$$

$$0.02 + 0.7W_w = 0.168$$

$$0.7W_w = 0.168 - 0.02$$

$$0.7W_w = 0.148.$$

$$W_w = 0.148 / 0.7 = 0.21\text{m}. \quad (54)$$

$$H = L = 2.5W_w = 2.5 * 0.21 = 0.525\text{m} \quad (55)$$

$$A_w = H * W_w = W_w * L = 0.21 * 0.525 = 0.11\text{m}^2. \quad (56)$$

Therefore,

$$\text{total window area} = 2A_w \quad (57)$$

$$\text{or } 2W_w * L = (2 * 0.11)\text{m}^2 = \mathbf{0.22\text{m}^2}.$$

Entire area = Area of window + Area of yoke and core

$$= (0.22 + 0.168) = 0.388 = \mathbf{0.39\text{m}^2}. \quad (58)$$

Taking area covered by the insulator into consideration, and assuming the coil did not torch the two outer limbs, and then we have,

Insulation thickness = 2mm

$$L_c = L - 4\text{mm} = 0.525\text{m} - 0.004\text{m} = \mathbf{0.521\text{m}} \quad (59)$$

$$W_c = W_w - 2\text{mm}, W_c = 0.21\text{m} - 0.002\text{m} = \mathbf{0.208\text{m}} \quad (60)$$

$$\text{Area}_c = W_c * L_c = 0.51 * 0.208 = \mathbf{0.108\text{m}^2}. \quad (61)$$

**Total window area when insulation thickness is subtracted**

$$\text{Total Area}_c = 2\text{Area}_c = 2W_c * L_c = 2 * 0.108 = \mathbf{0.216\text{m}^2} \quad (62)$$

$$\text{Entire area} = \text{Area of window} + \text{area of yoke and core} \quad (63)$$

$$= (0.216 + 0.168) = 0.384 = \mathbf{0.38\text{m}^2}.$$

**Area of copper in the window or coil used  $A_{cu}$  is obtained as follows;**

$$A_{cu-\text{total}} = a_1 N_1 + a_2 N_2 = (I_1 N_1 / J) + (I_2 N_2 / J). \quad (64)$$

From calculation we already know the following,

$$N_1 = 18 \text{ turns}, N_2 = 2 \text{ turns}, J = 1064 \text{ Ampere m}^{-2}, I_1 = 2.56\text{A}, I_2 = 31.25\text{A}$$

Primary coil area;

$$A_{cu1} = I_1 N_1 / J = (2.56 * 18) / 1064 \quad (65)$$

$$= 46.08 / 1064 = \mathbf{0.0433\text{m}^2}$$

Secondary coil area;

$$A_{cu2} = I_2 N_2 / J = (31.25 * 2) / 1064 \quad (66)$$

$$= 62.5 / 1064 = \mathbf{0.058m^2}$$

Total coil area

$$A_{cu-total} = a_1 N_1 + a_2 N_2 = (I_1 N_1 / J) + (I_2 N_2 / J).$$

$$0.0433m^2 + 0.058m^2 = \mathbf{0.102m^2}. \quad (67)$$

**Diameter of the coil used in the primary winding is obtained as follows**

$$A_{cu1} = \pi d^2 / 4; \gg 0.0433 = 3.142d^2 / 4; \gg 0.1732 = 3.142d^2;$$

Therefore;

$$d = \sqrt{(0.1732 / 3.142)} = \sqrt{0.0551} \quad (68)$$

$$= \mathbf{0.2348m = 235mm \text{ or } 23.5cm}$$

**i.e 0.2m – approximately.**

**Diameter of the coil used in the secondary winding is obtained as follows**

$$A_{cu2} = \pi d^2 / 4; \gg 0.058 = 3.142d^2 / 4; \gg 0.2348 = 3.142d^2$$

Therefore;

$$d = \sqrt{(0.2348 / 3.142)} = \sqrt{0.0747} \quad (69)$$

$$= \mathbf{0.2734m = 273mm \text{ or } 27.3cm}.$$

**i.e 0.3m – approximately.**

Total window area by calculation result obtained previously taking insulation thickness into consideration;

$$A_{w-total} = 0.216m^2, A_{cu-total} = 0.102m^2$$

Actual window space in this design

$$K_w = A_{cu-total} / A_{w-total} = 0.102 / 0.216 = \mathbf{0.47} \quad (70)$$

= **0.5 – approximately.**

**Mean length of coil used in this design is calculated as follows;**

$$A_i = 0.14m^2 \text{ – obtained previously in equation} \quad (43)$$

Circumference of the core is approximated to  $\pi d$

$$\text{Area} = 0.56d^2 \text{ – cruciform section core.}$$

$$\text{Therefore; } A_i = \pi d; \gg 0.14 = 0.56d^2; \gg d^2 = 0.14 / 0.56; \gg$$

$$d = \sqrt{0.25}, = \mathbf{0.5m} \text{ –}$$

Therefore; mean length of turns,

For; secondary side,

$$\pi d * 2 \text{ turns} = 3.142 * 0.5 * 2 = \mathbf{3.142m}, \quad (71)$$

**i.e 3m – approximately**

For; primary side,

$$\pi d * 18 \text{ turns} = 3.142 * 0.5 * 18 = \mathbf{28.278m}, \quad (72)$$

**i.e 28m – approximately.**

**Current drawn from the mains by the primary winding is obtained from,**

$$P_{rated} = I_1 V_1 \cos \Phi, \quad (73)$$

$$I_1 = P / V_1 \cos \Phi = 450 / 220 * 0.8 = \mathbf{2.56A}.$$

**Current to be delivered to load at the secondary terminal is obtained from,**

$$P_{rated} = I_2 V_2 \cos \Phi, \quad (74)$$

$$I_2 = P / V_2 \cos \Phi = 450 / 18 * 0.8 = \mathbf{31.25A}$$

**Therefore power rating of the transformer is obtained as follows**

$$P_{out-rated} = V_2 I_2 \cos \Phi = 18 * 31.25 * 0.8 \quad (75)$$

$$= \mathbf{450W.} \text{ – output power rating}$$

$$P_{in-rated} = V_1 I_1 \cos \Phi = 220 * 2.56 * 0.8 \quad (76)$$

$$= \mathbf{450.56W.} \text{ – input power rating.}$$

$$\text{Power loss} = P_{in-rated} - P_{out-rated} \quad (77)$$

$$= 450.56 - 450 = \mathbf{0.56W.}$$

**Rated Power**

Applying equation .....

$$S = 2.22 F B_m A_i J A_w K_w, \quad (78)$$

we have  $F = 50\text{Hz}$ ,  $B_m = 0.42 \text{ tesla}$ ,  $A_i = 0.14m^2$ ,  $\Phi_m = 0.059 \text{ Weber}$

$A_w = 0.216m^2$  (insulation thickness subtracted)

$K_w = 0.3$  – Window space factor

$J = ?$ ,  $S = P_{rated} = 450W$

$$J = S_{rated} / 2.22 F B_m A_i A_w K_w$$

$$= 450 / (2.22 * 50 * 0.42 * 0.14 * 0.216 * 0.3)$$

$$J = 450 / 0.42293664 = 1063.99 = \mathbf{1064 \text{ Amperem}^{-2}}. \quad (79)$$

Therefore  $P_{rated} = S_{rated} = 2.22 F B_m A_i J A_w K_w$

$$P_{rated} = S_{rated} = 2.22 * 50 * 0.42 * 0.14 * 1064 * 0.216 * 0.3 = 450.00458$$

$$P_{rated} = S_{rated} = \mathbf{450Watt}. \quad (80)$$

**Efficiency = power output/power input \*100**

$$\text{Power input} = P_{in} = I_1 V_1 \cos \Phi \quad (81)$$

$$= 2.56 * 220 * 0.8 = \mathbf{450.56W}$$

$$\text{Power output} = P_{out} = I_2 V_2 \cos \Phi \quad (82)$$

$$= 31.25 * 18 * 0.8 = \mathbf{450W}$$

$$\text{Efficiency} = (450 / 450.56) * 100 = \mathbf{99.88\%}. \quad (83)$$

## 18. Conclusion

This manuscript presented design principles of power transformer (VA-transformer), laying bare the basic design calculations required to design a functional transformer. The unique nature of this paper is seen in its simplicity and straight forwardness. It broke down transformer design process to easily understandable form and removed ambiguity usually introduced by complex computational techniques found in most design paper. Readers with average knowledge as well as professionals would find this paper a handy guide.

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