

# Throughput Enhancement via Iterative Optimization Approach and Modifying Max-SINR Algorithm for the MIMO Interference Network under Imperfect Channel State Information

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**Abstract** Design of robust transceiver for data rate improvement in interference channel (IC), under imperfect channel state information (CSI), is an important research area. This paper, employs an iterative optimization approach to design algorithm for throughput enhancement in a multi-input multi-output (MIMO) IC. Nodes in the MIMO IC, work in a time division duplex mode, where half of them are equipped with  $M$  antennas while the others have  $N$  antennas. In the proposed scheme, each transceiver adjusts its associated filter based on the maximization of the signal-to-interference-plus-noise ratio (SINR). In the time division duplex working mode, the problem utilizes reciprocity of the wireless network. Furthermore, it is investigated how the algorithms proposed by Gomadam et al. can be modified to enhance throughput under CSI error. With the knowledge of error variance Max-SINR is modified. Simulation results present the throughput performances of the proposed algorithms.

**Keywords:** imperfect channel state information, MIMO, interference channel, Max-SINR, sum rate

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## 1. Introduction

To date, different approaches have been developed to address interference management. Beside the conventional methods for interference management [1], a new method termed "interference alignment" (IA) has been proposed by the researchers. The basic idea behind the IA is to fit undesirable signals into a small portion of the signal space, observed by each receiver (interference subspace), and then leave the remaining signal space free of any interference for the desired signal (signal subspace). In [1-13], the authors implement the IA for different scenarios. The performance of the IA scheme is sensitive to the inaccuracy in channel state information (CSI). Many studies have already been carried out on performance analysis or improvement of the sum rate of the IA under CSI uncertainties. For example, capacity analysis can be found in [14]. Some approaches have employed feedback strategies to improve the sum rate [15,16,17,18].

This study includes two major parts. In the first part of this paper, throughput improvement of the algorithms proposed by Gomadam et al. [4], under the CSI error is investigated. Transceiver for multi-input multi-output (MIMO) interference channel (IC) has been designed by progressive minimization of the leakage interference

[4], Algorithm 1]. In this scheme, the IA is achieved only at very high SNRs. The Max-SINR algorithm [4], Algorithm 2] is another approach to obtain IA. This technique shows significant improvements in terms of sum rate in the range of low-to-intermediate SNRs and achieves the IA at high SNR. Some literature particularly focused on the performance analysis of algorithms. For example, the convergence issue of Max-SINR has been addressed in [20]. Throughput performances of these transceivers are sensitive to the CSI error. Algorithm performance is limited after a certain SNR and saturates [14]. However, performance improvement of these schemes under the CSI error has not been seriously considered so far. In this paper, the Max-SINR is modified to improve the sum rate subject to the CSI uncertainties. Receive filter consist of some parameters. The idea is that parameters are approximated by means over error. New receive filter is computed with respect to new parameters.

Other transceiver has been designed by minimization of the mean square error [10]. Also, mean square error criterion is averaged over error to improve sum rate of the MIMO interference network under imperfect CSI [19]. In the second proposed algorithm, iterative optimization approach is utilized to design beamformer based on the interference alignment. Each transceiver adjusts its transmit/receive filter by maximizing the SINR degraded by imperfect CSI. Simulation results demonstrate the

proposed algorithm achieves better sum rate performance compared with minimization of the mean square error [10] and the ones averaged over error [19]. The cost for better sum rate performance is the complexity.

The authors in [21] proposed a robust distributed joint signal and interference alignment design for the MIMO cognitive radio networks. Robust precoder and decorrelator were proposed in [22] and [23] for the multi-input single-output (MISO) and MIMO broadcast systems, respectively.

The remaining sections of this paper are organized as follows. Section 2 presents the system model. In section 3, a modified Max-SINR algorithm is proposed which enforces robustness against the imperfect CSI. The iterative optimization approach to design robust transceiver for the MIMO interference channel is explained in Section 4. Simulation results are presented in section 5 and concluding remarks are summarized in section 6.

## 2. System Model

In a  $K$ -user MIMO interference channel (IC), transmitter  $j$  and receiver  $k$  has  $M$  and  $N$  antennas, respectively. Index  $j \in \mathcal{K}$  is used to designate the transmitter, where  $\mathcal{K} = \{1, \dots, K\}$  and index  $k \in \mathcal{K}$  is used to denote the receiver. The  $j^{\text{th}}$  transmitter sends symbol vector  $s^j = [s_1^j \dots s_D^j]^T$  to the target receiver. Vector contains  $D$  independent symbols each of power  $P$ .

True and estimated channel matrices between transmitter  $j$  and receiver  $k$  are denoted by  $H^{kj}$  and  $\widehat{H}^{kj}$ , respectively. The error model is described by (1). The elements of  $\Delta H^{kj}$ , error matrix, are independent and identically distributed (i.i.d.) zero mean Gaussian of variance  $\sigma^2$ . All matrices are of dimension  $N \times M$ .

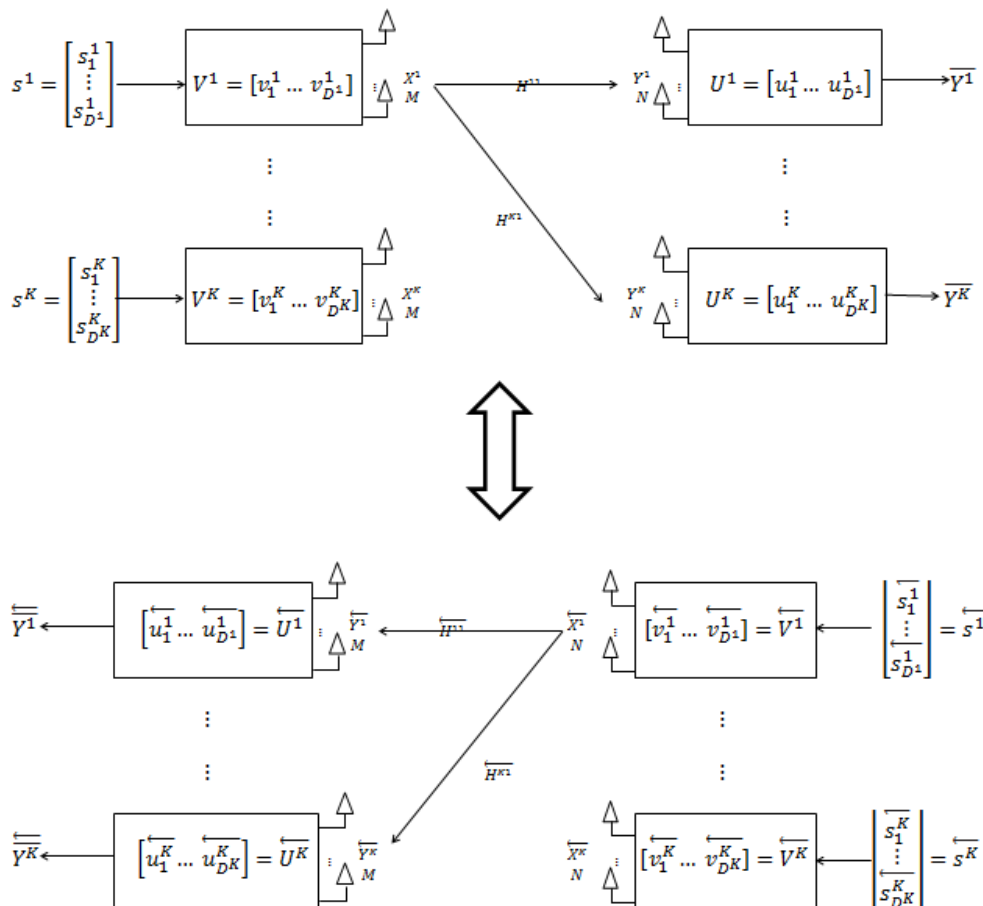
$$H^{kj} = \widehat{H}^{kj} + \Delta H^{kj}. \tag{1}$$

The received signal at receiver  $k$  is expressed by:

$$Y^k = \sum_{j=1}^K (\widehat{H}^{kj} + \Delta H^{kj}) X^j + Z^k \tag{2}$$

where,  $X^j$  is the  $M \times 1$  signal vector transmitted by the transmitter  $j$  and  $Z^k \sim \mathcal{N}(0, N_0 I)$  is the  $N \times 1$  additive white Gaussian noise (AWGN) vector. Beam-forming strategy is used based on the interference alignment. In particular, transmitter  $j$  precodes symbol vector by using the precoder matrix. Hence, the transmit signal can be expressed by  $X^j = V^j s^j$ , where  $V^j = [v_1^j \dots v_D^j]$  is the  $M \times D$  precoder matrix. Columns of  $V^j$ ,  $v_d^j$ , are unit norm vectors. Receiver  $k$  estimates the transmitted symbol vector  $s^k$  by using the interference suppression matrix  $U^k = [u_1^k \dots u_D^k]$  of dimension  $N \times D$ . The received signal is filtered by  $U^k$  as  $\overline{Y}^k = U^{k \dagger} Y^k$ .

Each node works in a time division duplex (TDD) mode. At two consecutive time slots, first, nodes on the left-hand side send the data to the nodes on the right-hand side. Then the role of nodes is switched and the nodes on the left-hand side receive the data, as illustrated in Figure 1.



**Figure 1.** System model. Reciprocal network (below channel) is obtained by switching the roles of transmitters and receivers in the original channel (top network). Original and reciprocal channels distinguish two working modes

To denote channels, filters, and etc. on the reciprocal channel, a left arrow is used on top of each notation. The relation between the original and reciprocal channel matrices is  $\overleftarrow{H}^{jk} = H^{kj \dagger}$  [4]. The operator  $(\cdot)^\dagger$  denotes the conjugate transpose of a matrix. Since the receivers of the reciprocal channel play the role of original network's transmitters and vice versa, one can write  $\overleftarrow{V}^k = U^k$  and  $\overleftarrow{U}^j = V^j$ .

The SINR degraded by imperfect CSI for the  $d^{th}$  data stream at  $k^{th}$  receiver [[24], Appendix A] is as follow.

$$SINR_d^k(h^k) = \frac{P u_d^{k \dagger} \widehat{H}^{kk} v_d^{k2} - P h^{kk} u_d^{k2}}{\left( \begin{array}{l} P \sum_{j=1}^K \sum_{m=1}^D \left\| u_d^{k \dagger} \widehat{H}^{kj} v_m^j \right\|^2 \\ + P D \left\| u_d^k \right\|^2 \sum_{j=1}^K h^{kj} - P \left\| u_d^{k \dagger} \widehat{H}^{kk} v_d^k \right\|^2 \\ - P h^{kk} \left\| u_d^k \right\|^2 + N_0 \left\| u_d^k \right\|^2 \end{array} \right)}, \quad (3)$$

where  $\|\cdot\|^2$  denotes Frobenius norm.  $SINR_d^k(h^k)$  is a random variable and  $h^k$  is a random vector,  $h^k = [h^{k1} \dots h^{kk}]^t$ .  $h^{kj}$  denotes norm of error matrix between transmitter  $j$  and receiver  $k$ ,  $h^{kj} = \|\Delta H^{kj}\|^2$ .

## 2.1. Statistic of $h^{kj}$

Normalized norm has a Chi-square distribution with  $2MN$  degrees of freedom  $\frac{h^{kj}}{\sigma^2/2} \sim \chi_{2MN}^2$ , also we have  $E[h^{kj}] = MN\sigma^2$ , and  $VAR[h^{kj}] = MN\sigma^4$  [25]. Accordingly, the expected value of the random vector can be expressed by  $\rho = E[h^k] = [MN\sigma^2 \dots MN\sigma^2]^t$  and the covariance matrix is

$$Cov(h^k) = \begin{bmatrix} MN\sigma^4 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & MN\sigma^4 \end{bmatrix}.$$

## 3. Modified Max-SINR Algorithm

In this section, the throughput improvement of the algorithms developed by Gomadam et al. under the CSI error is considered. The receive interference suppression matrix of the Max-SINR algorithm (equation 31 in [4]) is expressed by,

$$u_d^k = \frac{(B_d^k + N_0 I)^{-1} H^{kk} v_d^k}{\left\| (B_d^k + N_0 I)^{-1} H^{kk} v_d^k \right\|}. \quad (4)$$

Interference matrix for the  $d^{th}$  data stream at the  $k^{th}$  receiver is  $B_d^k$ .

$$B_d^k = P \sum_{j=1}^K \sum_{m=1}^D \left( \widehat{H}^{kj} + \Delta H^{kj} \right) v_m^j v_m^{j \dagger} \left( \widehat{H}^{kj} + \Delta H^{kj} \right)^\dagger - P \left( \widehat{H}^{kk} + \Delta H^{kk} \right) v_d^k v_d^{k \dagger} \left( \widehat{H}^{kk} + \Delta H^{kk} \right)^\dagger. \quad (5)$$

It is seen that  $u_d^k$  is a function of error matrices. Next, the mean of  $B_d^k$ , conditioned on  $\widehat{H}^{kj}$ , is computed as follow

$$\mu = E \left[ B_d^k \mid \widehat{H}^{kj} \right] = W^k - L_d^k + P\sigma^2(KD-1)I, \quad (6)$$

where, we have used

$$E \left[ \Delta H^{kj} v_m^j v_m^{j \dagger} \Delta H^{kj \dagger} \right] = \sigma^2 \left( v_m^j v_m^{j \dagger} \right) I = \sigma^2 I.$$

In (6),  $W^k = P \sum_{j=1}^K \sum_{m=1}^D \widehat{H}^{kj} v_m^j v_m^{j \dagger} \widehat{H}^{kj \dagger}$  and  $L_d^k = P \widehat{H}^{kk} v_d^k v_d^{k \dagger} \widehat{H}^{kk \dagger}$  denote the estimated covariance matrix of all data streams seen by the receiver  $k$  and estimated covariance matrix of  $d^{th}$  desired data stream.

In this way,  $B_d^k$  can be approximated by  $\mu$ . Therefore, the receive filter of Max-SINR algorithm with respect to  $\mu$  is given by

$$u_d^k \cong \frac{\left( W^k - L_d^k + (P\sigma^2 KD - P\sigma^2 + N_0)I \right)^{-1} \widehat{H}^{kk} v_d^k}{\left\| \left( W^k - L_d^k + (P\sigma^2 KD - P\sigma^2 + N_0)I \right)^{-1} \widehat{H}^{kk} v_d^k \right\|}. \quad (7)$$

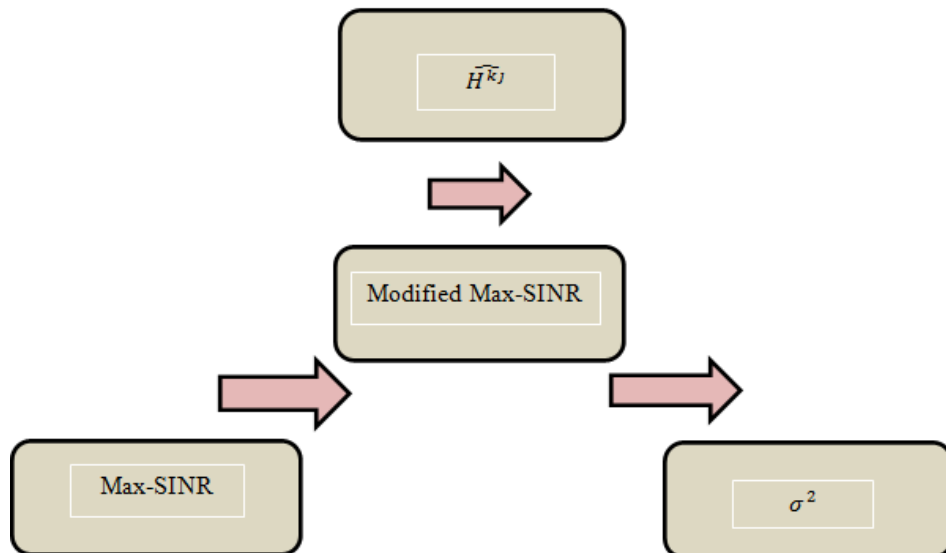


Figure 2. Modified MAX-SINR algorithm

The proposed filter, which incorporates error variance into formulation, can improve the sum rate performance of the Max-SINR algorithm under the CSI error, Figure 2.

If we follow the similar approach as followed in case of Max-SINR, leakage minimization algorithm does not give improvement for imperfect CSI. The  $d^k$ -dimensional received signal subspace that contains the least interference is the space spanned by the eigenvectors corresponding to the  $d^k$  smallest eigenvalues of the interference covariance matrix  $Q$ . Thus, the  $D$  columns of  $U^k$  are given by (equation 22 in [4])

$$u_d^k = \vartheta_d [Q], \forall d \in \{1, \dots, D\},$$

$$Q = P \sum_{j=1}^K \sum_{m=1}^D \left( \widehat{H}^{kj} + \Delta H^{kj} \right) v_m^j v_m^{j\dagger} \left( \widehat{H}^{kj} + \Delta H^{kj} \right)^\dagger, \quad (8)$$

where,  $\vartheta_d [Q]$  is the eigenvector corresponding to the  $d^{th}$  smallest eigenvalue of  $Q$ .

Since, only channel estimates  $\widehat{H}^{kj}$  are available, the interference covariance matrix can be approximated by  $E[Q]$ .

$$Q \cong E[Q] = P \sum_{j=1}^K \sum_{m=1}^D \widehat{H}^{kj} v_m^j v_m^{j\dagger} \widehat{H}^{kj\dagger} + (P\sigma^2 (K-1)D)I. \quad (9)$$

In this way, the  $d^{th}$  smallest eigenvector of  $E[Q]$  is

$$u_d^k = \vartheta_d [E[Q]] = \vartheta_d \left[ \sum_{j=1}^K \sum_{m=1}^D \widehat{H}^{kj} v_m^j v_m^{j\dagger} \widehat{H}^{kj\dagger} \right]. \quad (10)$$

In spite of Max-SINR algorithm, substituting the columns of  $U^k$  of the Leakage minimization algorithm by the eigenvectors of the approximated  $Q$  cannot modify it with improved sum rate performance. In fact, new eigenvectors are just based on the CSI estimates.

## 4. Iterative Optimization Approach

In this section, an algorithm is designed to achieve the robust transceiver for the MIMO IC. The goal is to achieve a robust transceiver by iteratively updating transmit and receive filters in order to increase the SINR degraded by imperfect CSI. The iterative algorithm alternates between the original and reciprocal networks. Inside each network, only the filters associated with the receivers are updated.

Step I: In the original network, each receiver solves the following optimization problem.

$$\max_{u_d^k} SINR_d^k, \forall d \in \{1, \dots, D\}. \quad (11)$$

Step II: In the reciprocal network, the following problem in (12) is solved with the fixed transmit precoding matrices. The matrices are receive interference suppression filters from original network, already determined

in Step I. Each receiver updates its columns of interference suppression filter as follow:

$$\max_{u_d^k} \overline{SINR_d^k}, \forall d \in \{1, \dots, D\}. \quad (12)$$

Then, the receive interference suppression filters in the reciprocal network are used as the fixed matrices in step I. The above-mentioned steps are iterated until the algorithm converges.

### 4.1. Estimating the Mean of $SINR_d^k$

Considering the covariance matrix form,  $Cov(h^k) = \begin{bmatrix} MN\sigma^4 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & MN\sigma^4 \end{bmatrix}$ , the probability density function (PDF) of  $h^k$ ,  $f(h^k)$ , is concentrated around its mean. Therefore,  $E[SINR_d^k(h^k)]$  can be expressed in terms of mean vector  $\rho$ . By using the statistical linearization argument [26], the  $SINR_d^k$  is approximated by a first order Taylor series expansion around the mean value,  $\rho$ , to yield:

$$SINR_d^k \cong SINR_d^k(\rho) + \sum_{j=1}^K \frac{\partial SINR_d^k(\rho)}{\partial h^{kj}} (h^{kj} - MN\sigma^2). \quad (13)$$

In this case

$$E[SINR_d^k(h^k)] \cong SINR_d^k(\rho) + \sum_{j=1}^K \int \frac{\partial SINR_d^k(\rho)}{\partial h^{kj}} (h^{kj} - MN\sigma^2) f(h^k) dh^k. \quad (14)$$

The output of the integrations are zero,  $\int h^{kj} f(h^k) dh^k = E[h^{kj}] = MN\sigma^2 = MN\sigma^2 \int f(h^k) dh^k$ , hence estimation of the mean value results in  $E[SINR_d^k] \cong SINR_d^k(\rho)$  where,

$$E[SINR_d^k | \widehat{H}^{kj}] \cong \frac{u_d^{k\dagger} [L_d^k - PMN\sigma^2 I] u_d^k}{u_d^{k\dagger} [W^k - L_d^k + (PMN\sigma^2 KD - PMN\sigma^2 + N_0)I] u_d^k} \quad (15)$$

In (15),  $W^k = P \sum_{j=1}^K \sum_{m=1}^D \widehat{H}^{kj} v_m^j v_m^{j\dagger} \widehat{H}^{kj\dagger}$  and  $L_d^k = P \widehat{H}^{dk} v_d^k v_d^{k\dagger} \widehat{H}^{dk\dagger}$  are appeared as in (6).

### 4.2. Explanation about $E[SINR_d^k]$

The estimation in (15) decreases with increase in P, an unexpected behavior. Higher order Taylor series expansions can lead to a better approximation but more complex. In (15), unwanted signals from other users are fitted into interference subspace.

$$u_d^{k\dagger} (W^k - L_d^k) u_d^k : \text{Leakage of unwanted signals}$$

$$PMN\sigma^2 (KD-1) : \text{Added Leakage due to imperfect CSI.}$$

The signal subspace is left free of interference for the desirable signal.

$$u_d^{k\dagger} L_d^k u_d^k : \text{desirable signal}$$

$MN\sigma^2$  : Reduced power due to imperfect CSI.

Naturally, it is reasonable to consider

$$\text{Added Leakage} < \text{Leakage}.$$

It is possible inequality does not hold mathematically. Consequentially,  $E[SINR_d^k]$  decreases with increase in P, unexpected behavior. Introducing  $0 < \alpha < \frac{u_d^{k\dagger} (W^k - L_d^k) u_d^k}{PMN\sigma^2(KD-1)}$  as a weight for added leakage can prevent such behavior:

$$E[SINR_d^k | H^{kj}] \cong \frac{u_d^{k\dagger} [L_d^k - PMN\sigma^2 I] u_d^k}{u_d^{k\dagger} [W^k - L_d^k + (\alpha PMN\sigma^2 (KD-1) + N_0) I] u_d^k}.$$

### 4.3. Iterative Solution

Maximizing (15) over  $u_d^k$  can be written as follow

$$\max \frac{u_d^{k\dagger} G u_d^k}{u_d^{k\dagger} F u_d^k}, \tag{16}$$

where matrices are  $G = G^\dagger = L_d^k - PMN\sigma^2 I \geq 0$ , and  $F = F^\dagger = W^k - L_d^k + (PMN\sigma^2 KD - PMN\sigma^2 + N_0) I > 0$ . It is shown in [27] that the optimization problem in (16) is equivalent to

$$\max u_d^{k\dagger} G u_d^k, \text{ s. t. } u_d^{k\dagger} F u_d^k = 1. \tag{17}$$

For the equivalent problem, i.e. constrained maximization in (17), Lagrangian function is  $l(u_d^k, \lambda) = u_d^{k\dagger} G u_d^k + \lambda (1 - u_d^{k\dagger} F u_d^k)$ . Lagrange conditions are  $\frac{\partial l(u_d^k, \lambda)}{\partial u_d^k} = \mathbf{0}$  and  $\frac{\partial l(u_d^k, \lambda)}{\partial \lambda} = 0$ . The solution is denoted by  $u_d^{k*}$  and Lagrange multiplier by  $\lambda^*$ . It is also shown in [27] that  $u_d^{k*}$  is the eigenvector corresponding to the maximal eigenvalue of  $F^{-1}G$  and  $\lambda^*$  is  $u_d^{k*\dagger} G u_d^{k*}$ .

Therefore, the unit vector that maximizes (15), is given by

$$u_d^k = \vartheta[F^{-1}G], \tag{18}$$

where operator  $\vartheta[\cdot]$  denotes the eigenvector corresponding to the maximal eigenvalue of a matrix. Now, we consider the reciprocal network. The transmit precoding matrices,  $\bar{V}^k$ , are the receive interference suppression matrices  $U^k$  from the original network that their columns are given by (18). The optimal  $d^{th}$  unit column of  $\bar{U}^j$ , is given by

$$\bar{u}_d^j = \vartheta[\bar{F}^{-1}\bar{G}]. \tag{19}$$

Now, receive interference suppression matrices in the reciprocal network, obtained using (19), are put in places

of transmit precoding matrices in the original network, and new receive filters are determined accordingly. The switching between both channels continues in this manner. The steps of the algorithm are given in Table 1.

**Table 1. Iterative Optimization Approach and schematic view**

Algorithm
Pick arbitrary precoding matrices $V^j$ of size $M \times D$ to initialize.
REPEAT
1: Compute $U^k$ in original channel: $u_d^k = \vartheta[F^{-1}G], \forall k \in \mathcal{K}, d \in \{1, \dots, D\}$ .
2: Set $\bar{V}^k = U^k \forall k \in \mathcal{K}$ .
3: Compute $\bar{U}^j$ in reciprocal channel: $\bar{u}_d^j = \vartheta[\bar{F}^{-1}\bar{G}], \forall j \in \mathcal{K}, d \in \{1, \dots, D\}$ .
4: Set $V^j = \bar{U}^j \forall j \in \mathcal{K}$ .
UNTIL ALGORITHM CONVERGES

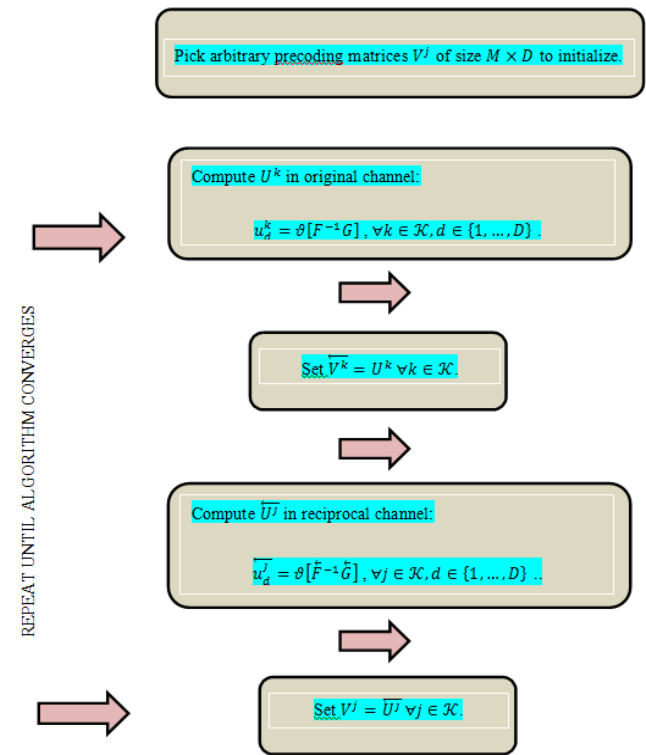


Figure 3.

### 4.4. Proof of Convergence

The convergence of the algorithm is proved by considering total Lagrangian function of all data streams in the network  $\sum_{k=1}^K \sum_{d=1}^D l(u_d^k, \lambda)$ . The metric is defined in (20). The function is unchanged in the original and reciprocal networks since the transmit and receive filters change their roles. Therefore, each step in the algorithm increases the value of the function. This implies that the algorithm converges.

$$\max_{\substack{V^j \text{ and } U^k \\ \forall j \text{ and } k \in \mathcal{K}}} \text{metric} = \sum_{k=1}^K \sum_{d=1}^D l(u_d^k, \lambda). \tag{20}$$



Accordingly:

$$\max_{\substack{U^k \\ \forall k \in \mathcal{K}}} \text{metric} = \sum_{k=1}^K \sum_{d=1}^D \max_{u_d^k} l(u_d^k, \lambda). \quad (21)$$

In other words, given  $V^j \forall j \in \mathcal{K}$ , Step 1 increases the value of (20) over all possible choices of  $U^k \forall k \in \mathcal{K}$ . The filter  $\bar{U}^j$  computed in Step 3, based on  $\bar{V}^k = U^k$ , also maximizes the metric in the reciprocal channel (22).

$$\max_{\substack{\bar{U}^j \\ \forall j \in \mathcal{K}}} \text{metric} = \sum_{j=1}^K \sum_{d=1}^D \bar{l}(\bar{u}_d^j, \bar{\lambda}). \quad (22)$$

Since  $\bar{V}^k = U^k$  and  $\bar{U}^j = V^j$ , the metric remains unchanged in the original and reciprocal networks. Therefore, Step 3 also can increase the value of (20). Since the value of (20) is monotonically increased after every iteration, convergence of the algorithm is guaranteed.

## 5. Simulation Results

The channel is modeled as Rayleigh flat fading. The channel coefficients, i.e. elements of the  $H^{kj}$  matrix, are i.i.d. zero mean unit variance Gaussian. All numerical results are averaged over error matrices. Averaging over error is repeated for several channels. Final numerical results are the average over repetitions.

### 5.1. Improved Performance of Modified Max-SINR

In this part, the improved sum rate performance achieved by the modified Max-SINR algorithm under CSI error is demonstrated. The throughput is given by  $R = \sum_{k=1}^K \sum_{d=1}^D \log(1 + \text{sinr}_d^k)$ .<sup>1</sup>

$$\begin{aligned} & \text{sinr}_d^k \\ &= \frac{P u_d^{k\dagger} \widehat{H}^{kk} v_d^{k2}}{P \sum_{j=1}^K \sum_{m=1}^D u_d^{k\dagger} \widehat{H}^{kj} v_m^{j2} - P u_d^{k\dagger} \widehat{H}^{kk} v_d^{k2} + N_0 u_d^{k2}}. \end{aligned} \quad (23)$$

Figure 4, Figure 5, and Figure 6 represent the sum rate comparison between algorithms for MIMO IC ( $K = 4, N = M = 3, D = 1$ ). The improved performance of Modified Max-SINR is demonstrated in these figures. They confirm that with the knowledge of error variance, the Max-SINR is modified to improve the sum rate under the CSI error. Figure 4 represents the sum rate comparison for error variance  $\sigma^2 = 0.025$ . In terms of sum rate improvement of modified Max-SINR, it has 4dB SNR gain over Max-SINR algorithm at 18 b/s/Hz sum data rate. About the proposed scheme in Table I presents better sum rate than the Modified algorithm in the SNR range between  $-5dB$  to  $16dB$ .

<sup>1</sup> For throughput computation in the SINR, imperfect channel estimate is used. The transmit precoding matrix and interference suppression matrix is computed on imperfect channel estimate and channel estimation error statistics.

The filters are designed with  $\sigma^2 = 0.05$  in Figure 5. For example, the Modified algorithm has 5dB SNR gain over Max-SINR algorithm at 16 b/s/Hz sum data rate. Algorithm in Table 1 performs more satisfactory compared to Max-SINR in the SNR range between  $-5dB$  to  $17dB$ .

Figure 6 shows the sum rate for  $\sigma^2 = 0.1$ . Figure 6 shows that the proposed algorithm in Table 1 produces sum data rate higher than the Leakage minimization, MMSE, and Robust MMSE.

Figure 7, and Figure 8 represent the sum rate comparison between algorithms for MIMO IC ( $K = 2, N = 3, M = 4, D = 2$ ). The filters are designed with error variance  $\sigma^2 = 0.025$ , and  $\sigma^2 = 0.05$ . Again, the improved performance of Modified algorithm is demonstrated in these figures. Proposed scheme in Table presents better sum rate than the Max-SINR for  $\sigma^2 = 0.025$ . Figure 8 shows that the proposed algorithm in Table 1 produces sum data rate as much as Max-SINR and higher than the Leakage minimization, MMSE, and Robust MMSE for  $\sigma^2 = 0.05$ .

The cost for better sum rate performance is the complexity since the MMSE, and Robust MMSE need the inverse operation of a matrix only once to update  $V^k$  (or  $U^k$ ) in each iteration, whereas the proposed algorithm require inverse operation  $D$  (number of independent data streams) times. In the SINR maximizing and modified algorithms, the transmit and receive filters are column-wise updated, require  $D$  inverse operation.

### 5.2. Ergodic Sum Rate

In this part, the improved performance of the algorithm in Table 1 is substantiated in terms of Ergodic sum rate degraded by imperfect CSI.<sup>2</sup>

$$\begin{aligned} E[C] &\cong \sum_{k=1}^K \sum_{d=1}^D \log(1 + E[\text{SINR}_d^k]) \\ &\cong \sum_{k=1}^K \sum_{d=1}^D \log \left( 1 + \frac{u_d^{k\dagger} [L_d^k - PMN\sigma^2 I] u_d^k}{u_d^{k\dagger} \left[ W^k - L_d^k + \begin{pmatrix} PMN\sigma^2 KD \\ -PMN\sigma^2 + N_0 \end{pmatrix} I \right] u_d^k} \right). \end{aligned} \quad (24)$$

A MIMO IC with four users,  $K = 4$ , and three antennas at the transmitters and receivers,  $N = M = 3$ , is considered. In this MIMO IC, each user transmits  $D = 1$  data stream. Figure 9 and Figure 10 represents the sum rate of the schemes when filters are designed with error variance  $\sigma^2 = 0.025$  and  $\sigma^2 = 0.05$ . It can be observed that proposed algorithm in Table 1 achieves higher sum rate than other schemes.

Figure 11 represents the sum rate when filters are designed with two error variances,  $\sigma_1^2 = 0.05$  and  $\sigma_2^2 = 0.1$ , for ( $K = 3, N = M = 2, D = 1$ ) MIMO IC. The sum rate of schemes are shown with dashed lines for  $\sigma_1^2 = 0.05$  and lines for  $\sigma_2^2 = 0.1$ . Superior sum rate performance of the proposed algorithm in Table 1 is obvious.

<sup>2</sup> For Ergodic sum rate in the signal power, imperfect channel estimate and Statistic of  $h^{kj}$  are used.

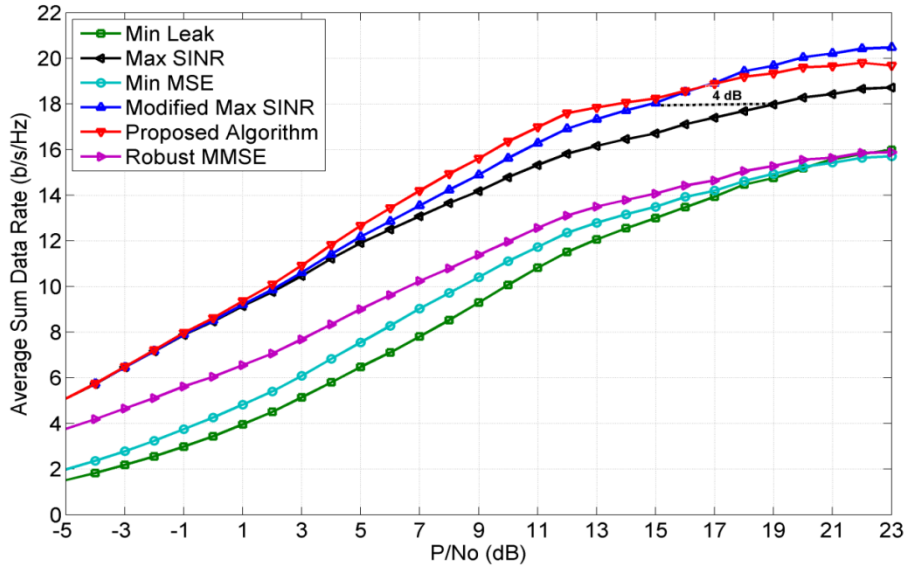


Figure 4. Average sum data rate versus SNR.  $K = 4, N = M = 3, D = 1, \sigma^2 = 0.025$ .

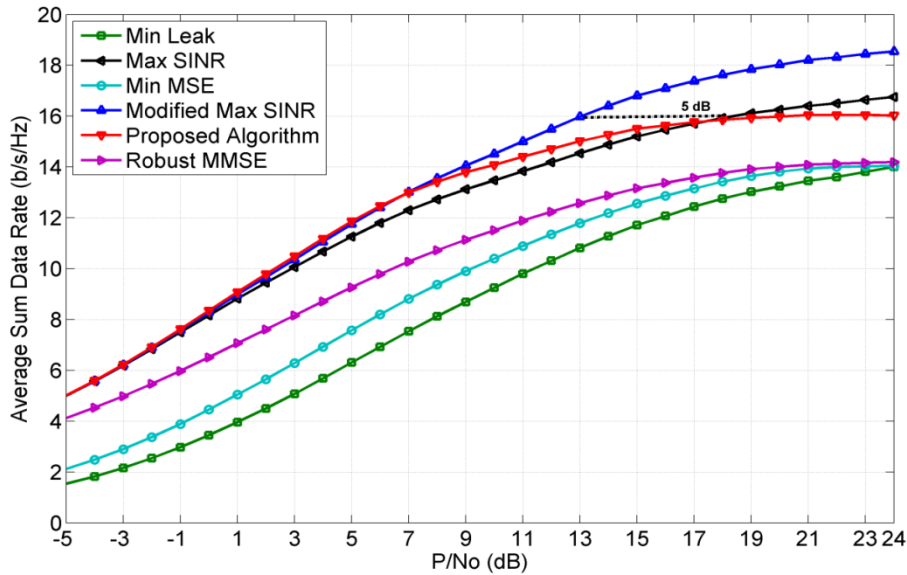


Figure 5. Average sum data rate versus SNR.  $K = 4, N = M = 3, D = 1, \sigma^2 = 0.05$

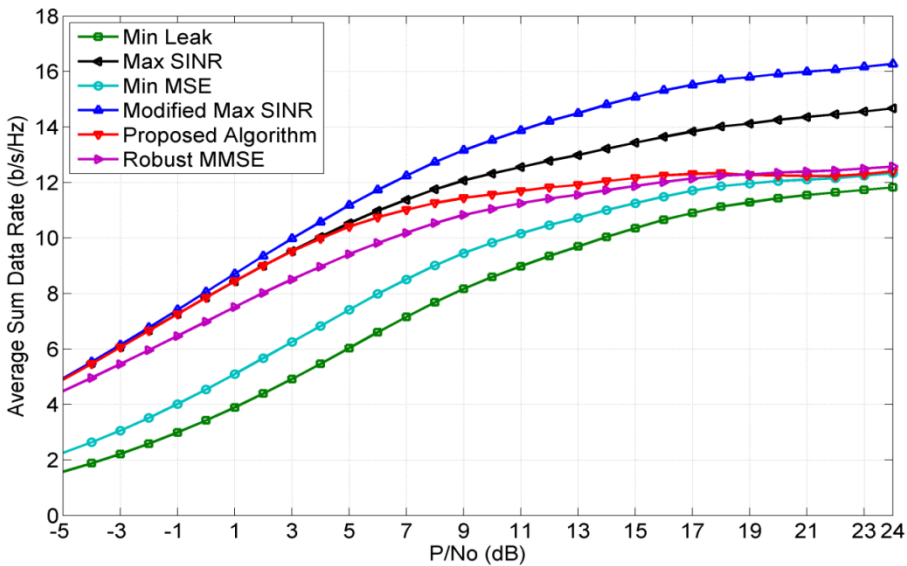


Figure 6. Average sum data rate versus SNR.  $K = 4, N = M = 3, D = 1, \sigma^2 = 0.1$ .

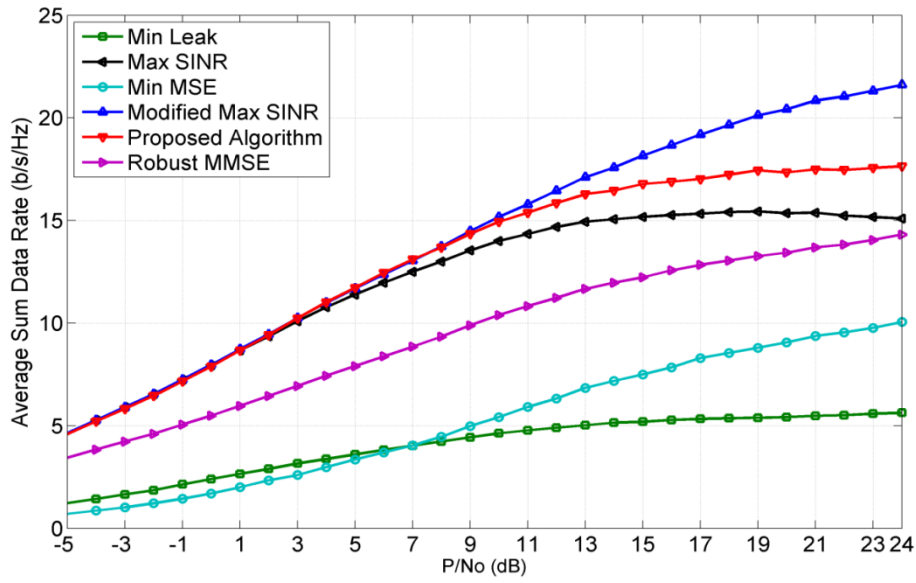


Figure 7. Average sum data rate versus SNR.  $K = 2, N = 3, M = 4, D = 2, \sigma^2 = 0.025$

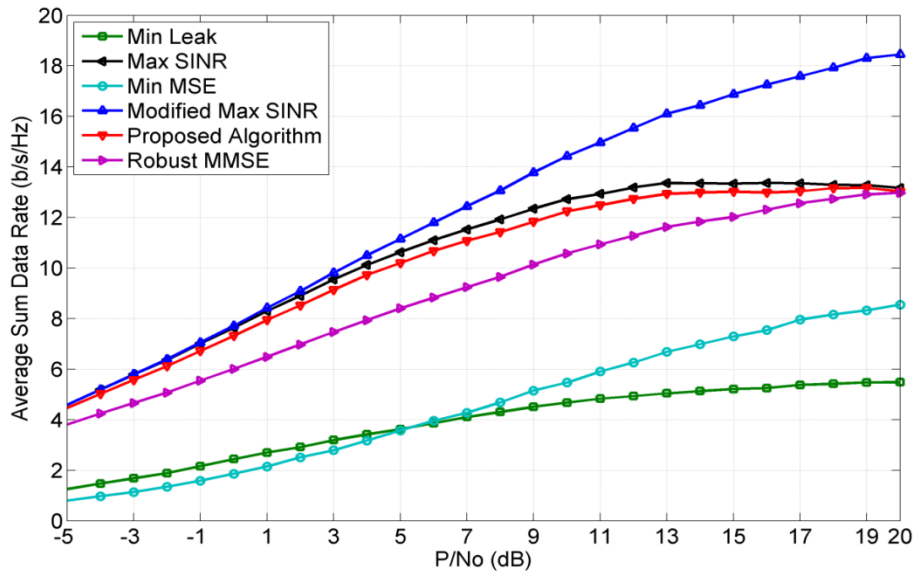


Figure 8. Average sum data rate versus SNR.  $K = 2, N = 3, M = 4, D = 2, \sigma^2 = 0.05$

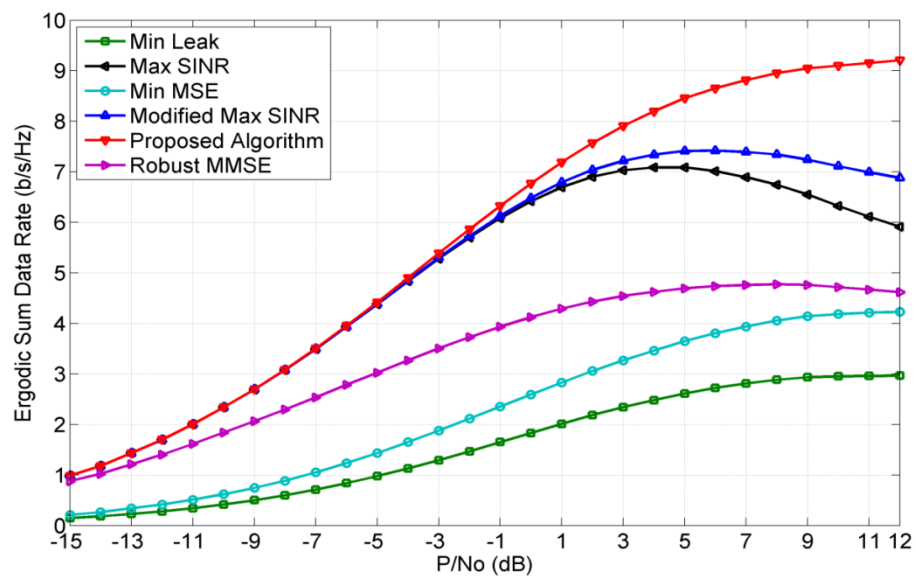


Figure 9. Average sum rate versus SNR.  $K = 4, N = M = 3, D = 1, \sigma^2 = 0.025$



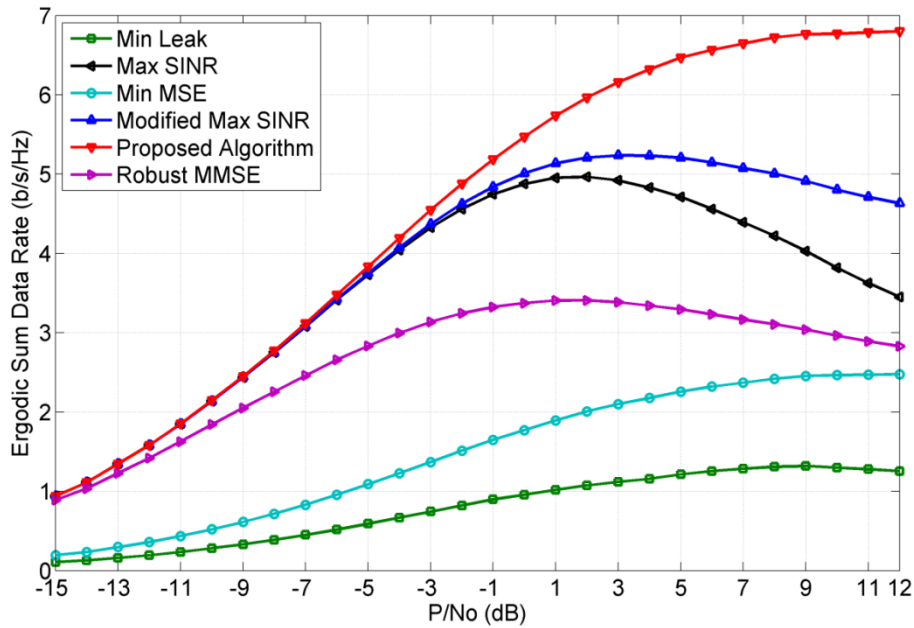


Figure 10. Average sum rate versus SNR.  $K = 4, N = M = 3, D = 1, \sigma^2 = 0.05$

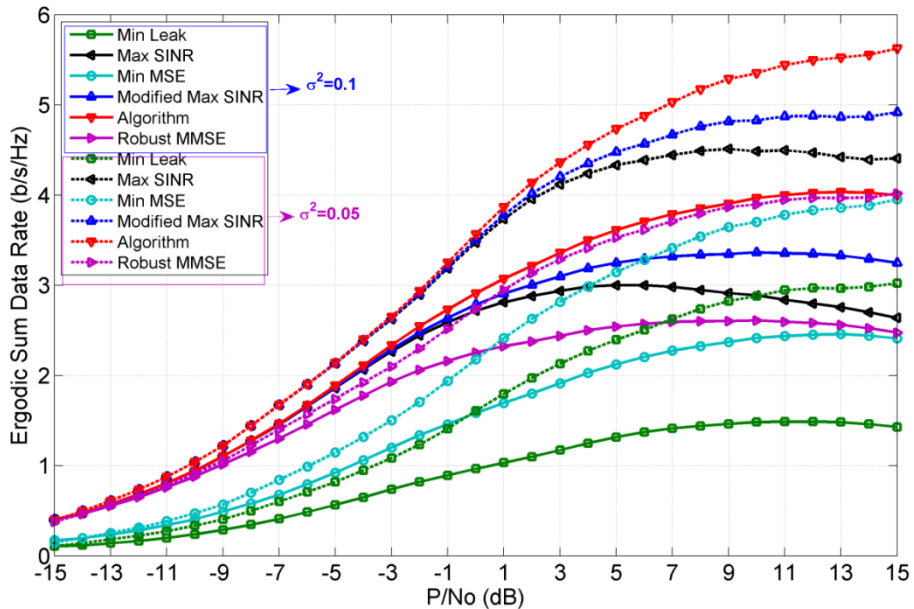


Figure 11. Average sum rate versus SNR.  $K = 3, N = M = 2, D = 1$

### 6. Conclusion

This paper talked about throughput enhancement in a MIMO interference channel under Imperfect CSI. The paper used inference alignment concept and proposed two algorithms. First algorithm, namely Modified Max-SINR is a modification on Max-SINR algorithm (proposed by Gomadamet.al.) to incorporate CSI error into account while designing transmit precoding matrix at transmitter and interference suppression matrix at receiver. Second algorithm design transmit precoding matrix and interference suppression matrix using an optimization approach by maximizing the SINR degraded by imperfect CSI at each receiver and transmitter. This approach is iterative in nature and assumes reciprocity of the wireless network.

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